PERGAMON

Deep-Sea Research I 47 (2000) 2317–2340

DEEP-SEA RESEARCH
PART I

Upwelling into the thermocline of the Pacific Ocean

Doron Nof[a,b,*], Stephen Van Gorder[a]

[a]Department of Oceanography 4320, Florida State University, Tallahassee, FL 32306-4320, USA
[b]Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306-4360, USA

Received 23 April 1999; received in revised form 13 September 1999; accepted 29 November 1999

Abstract

The established "island rule" and the recently introduced "separation formula" are combined to yield an analytical expression for the total upwelling into the thermocline in the Pacific. The combination of the two is achieved with the use of a hybrid model containing a stratified upper layer, a thick (slowly moving) homogenous intermediate layer and an inert lower layer. Both the upper and the intermediate layers are subject to diabatic cooling and heating (which need not be specified) and there is an exchange of mass between the two active layers. An attempt is made to examine the above analytical (hybrid) model numerically. Ideally, this should be done with a complete two-and-a-half layer model (with upwelling and downwelling), but such a model is much too complex for process-oriented studies (due to the required parameterization of vertical mixing). Consequently, we focus our attention on verifying that the separation formula and the island rule are consistent with each other in a much simpler, layer-and-a-half model (without upwelling). We first verified that the new "separation formula" provides a reasonable estimate of the wind-induced transport in an island-free basin. We then compare the wind-induced transport predicted by the separation formula and the island rule in an idealized basin containing an island. We show that in these idealized situations the two methods give results that are consistent with each other and the numerics. We then turned to an application of the (hybrid) two-and-a-half layer model to the Pacific where, in contrast to the idealized layer-and-a-half models (where the two methods address the same water mass), the two methods address two *different* water *masses*. While the separation formula addresses only thermocline water ($\sigma_\theta < 26.20$), the island rule addresses all the water down to $27.5\sigma_\theta$ (i.e., both the upper and intermediate layer). This is why the application of the two methods to the Pacific gives two different results — an application of the formula gives zero warm water transport whereas an

* Correspondence address: Department of Oceanography 4320, Florida State University, Tallahassee, FL 32306-4320, USA. Fax: 1-850-644-2581.

E-mail address: nof@ocean.fsu.edu (D. Nof).

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PII: S0967-0637(00)00029-7

Nomenclature

f	Coriolis parameter
g'	reduced gravity ($g\Delta\rho/\rho$)
k	linear drag coefficient
l	direction of the integration path
p	total pressure
P	derivation of hydrostatic pressure from the pressure associated with a state of rest
Q	total upwelling
T	angular momentum
T_{ind}	transport through the Indonesian Passages
u	zonal velocity
v	meridional velocity
V	vertically integrated transport
β	variation of the Coriolis parameter with latitude
ν	viscosity coefficient
ρ	upper layer density
τ	wind stress

application of the island rule gives 16 Sv. Namely, the difference between the amount predicted by the island rule (16 Sv) and the amount predicted by the separation formula (zero) enters the Pacific as intermediate water and is then somehow upwelled into the thermocline. The upwelling should take place north of the southern western boundary currents separation (40°S). © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Island rule; Separation formula; Upwelling; Thermocline

1. Introduction

The question of how much water upwells into the thermocline in the Pacific ($\sigma_\theta = 26.20$) is important to our understanding of the global oceanic circulation. The familiar concept regarding the great global conveyer is that the water that sinks in the North Atlantic and forms the North Atlantic Deep Water (NADW) is flowing horizontally to the Pacific and Indian Oceans, where it is gradually upwelled into the thermocline (Broecker, 1991; Munk, 1966; Stommel and Arons, 1960). In this scenario, the sinking is confined to localized regions (and, therefore, is rapid) whereas the upwelling is distributed over most of the ocean (and, therefore, is slow).

The upwelling compensates for the downward diffusion of heat; it is not a simple matter to calculate it because of our vague knowledge of the vertical eddy difficulty. One way that one can come up with some sort of an estimate is to take the NADW formation rate (15 Sv) and multiply it by the ratio of the Pacific surface area to the sum of the Pacific and Indian Ocean areas. This gives about 10 Sv which, according to the great global conveyor idea, should pass from the Pacific to the Indian Ocean via the Indonesian Passages. Observations have not converged yet on the average amount that goes through the Passages (see e.g., Lukas et al., 1996; Godfrey, 1996 and other articles in the same special JGR issue devoted to the throughflow), but 10 Sv appears to be a reasonable estimate. Much of the throughflow occurs in the upper 200 m above $26.20\sigma_\theta$ (see e.g., Wyrtki, 1961; Hautala et al., 1996), although there is some (not adequately documented) flow below that level. It appears that this deeper flow has a σ_θ less than 27.00 as the $27.00\sigma_\theta$ is the highest density water found in the Indonesian Seas (Hautala et al., 1996). This may well result from the fact that the sill in the Makassar strait is about 600 m.

Here, we argue that the mass flux upwelled into the thermocline in the Pacific can be computed from the wind field over the Pacific and the position of the western boundary current separation.

1.1. The analytical model

We shall construct a two-and-a-half layer analytical model consisting of a continuously stratified upper layer, a much thicker (and slowly moving) intermediate layer, and an infinitely deep lower layer (Fig. 1). Using this model, we shall show that the upwelling into the thermocline is the difference between the amount that the island rule says that the Indonesian Throughflow ought to be [16 Sv of water (lighter than $27.5\sigma_\theta$) in the absence of friction in the Indonesian passages] and the amount that the separation formula gives for the throughflow [zero Sverdrup of water (lighter than $26.2\sigma_\theta$) in the absence of upwelling]. We shall show that this is so because the island rule calculation corresponds to the sum of the transport in the upper and intermediate layer, whereas the separation formula corresponds to upper water alone. Although the results seem to be simple enough, they are not that easy to derive, and the reader should be prepared to address some fairly subtle issues related to the models.

In this context, it should be emphasized that the island rule is a *prognostic* relationship, giving the circulation around an island from the imposed wind stress and the geography alone. The separation formula, on the other hand, is a *diagnostic* relationship between the flow from the Pacific to the Indian Ocean, the position of the western boundary currents separation, the wind stress, and the geography. Also, it should be pointed out that both models require a level of no motion. The separation formula requires all waters below the surfacing thermocline ($26.20\sigma_\theta$ which is a few or several hundred meters) to have very small speeds, whereas the island rule requires the existence of a level of no motion somewhere above the main topographic features of the Pacific (say, 1500 m).

1.2. The numerical model

Ideally, our analytical model should be examined with a two-and-a-half layer numerical simulation, which allows for mass exchange between the upper layer and the intermediate layer. However, such a model requires the specification of various mixing parameterizations and, as such, is too complex for a process-oriented study of the kind that we are interested in. Consequently, we shall focus on a much simpler one-and-a-half layer model, where the separation formula and the island rule can be easily compared to each other (Fig. 2).

1.3. What is new in this work

The suggestion of upwelling is, of course, not new and neither is the recognition that the actual throughflow is a measure of the upwelling in the Pacific. [The latter follows

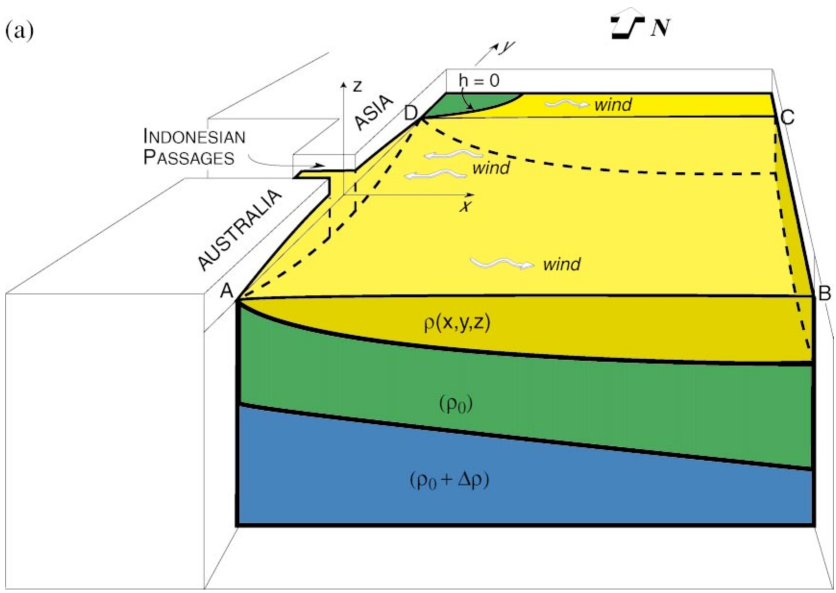

Fig. 1. (a) A schematic three-dimensional view of the conceptual two-and-a-half layer model under study. The upper layer (yellow) is continuously stratified and has a density $\rho(x, y, z)$; the interface separating it from the layer underneath outcrops. The second active (homogeneous) layer (green) is very thick so that its speeds (but not necessarily the transports) are negligible; its density is ρ_0. The third layer (blue) is inert and has no transport; its constant density is $\rho_0 + \Delta\rho$. Upper water (yellow) corresponds to the Indonesian transport captured by the separation formula ($\sigma_\theta \leqslant 26.20$), and the intermediate (green) water is the excess waters captured by the island rule ($26.20 \leqslant \sigma_\theta \leqslant 27.5$) but not by the formula. The difference between the two is upwelled into the thermocline in the Pacific. (b) A top view of the problem under study. The distance between the separating streamline ($h = 0$) and the western boundary of the Pacific Ocean ($x = 0$) is denoted by $\xi(y)$. The separation formula gives the transport of thermocline water ($\sigma_\theta < 26.20$) through the Indonesian passages, whereas the island rule gives the total amount of water (down to $27.5\sigma_\theta$) going through the passages. Since most of the water in the passages have $\sigma_\theta < 26.20$, we argue that the difference between the two is the upwelling into the thermocline in the Pacific. Note that in the derivation of the separation formula, it is assumed that the interface separating the (stratified) upper layer from the lower layer [whose speeds (but not necessarily the transports) are zero] outcrops ($h = 0$). The island rule, on the other hand, considers a (stratified) upper layer, which goes down all the way to a level of no motion above the topography.

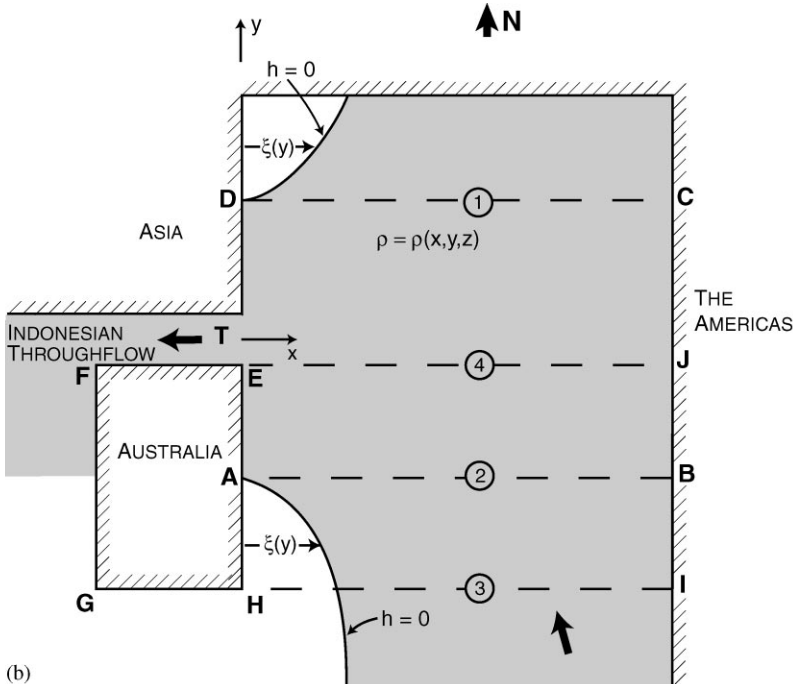

Fig. 1. (*Continued*).

from the separation formula, which states that no warm water ($\sigma_\theta < 26.20$) can be forced directly into the Indonesian Seas (Nof, 1998)]. What is new in this article is (i) the conceptual construction of the analytical two-and-a-half layer model, (ii) the verification of the separation formula numerically, (iii) the numerical comparison of the formula to the island rule, and (iv) the important recognition that both the separation formula and the island rule allow for density variations (due to mixing or diabatic processes) within their active layers. This latter recognition implies that, even though diabatic processes are not solved for in the models, they are an integral part of the dynamics and, consequently, the moving layers can have water mass conversion within them. It is the presence of this water mass conversion process that allows us to conclude that the difference between the two calculations reflects the upwelled amount of water.

Before proceeding and completing this introduction, we should probably state that we reached our last conclusion (that the difference between the island rule prediction and the separation formula prediction gives the upwelling in the Pacific) after a long period of discussion on how the newly derived separation formula can be reconciled with the more established island rule. The reader should be prepared to see some important differences between statements made in the present article and statements in earlier literature.

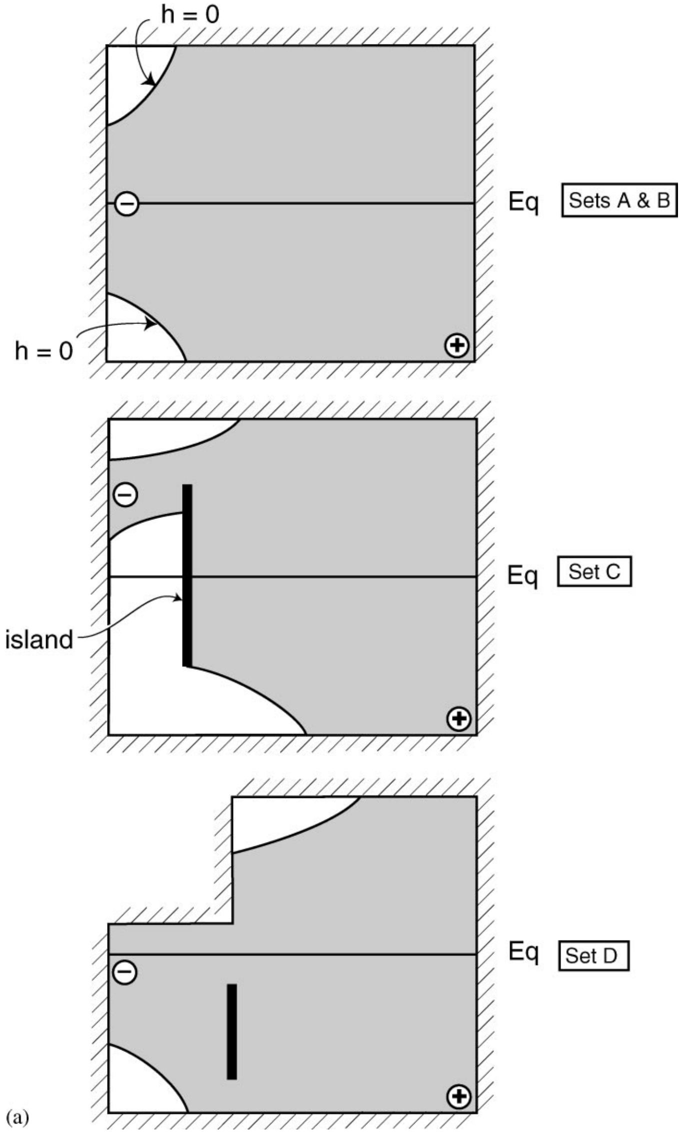

Fig. 2. (a) Schematic diagram of the layer-and-a-half numerical experiments corresponding to Sets A and B (upper panel), Set C (middle panel), and Set D (lower panel). A specified source–sink flow (denoted by $\oplus$ and $\ominus$) is superimposed on gyres driven by zonal winds. The transport predicted by the position of the western boundary current separation (i.e., the separation formula transport) is then compared to the specified value. Sets C and D contain an island. Shading denotes the upper layer. (b) Details of the source–sink numerical simulations and the magnified (and simplified) zonal wind stress as a function of the adjusted meridional scale [the magnified wind profiles were adapted from Hellerman and Rosenstein, 1983].

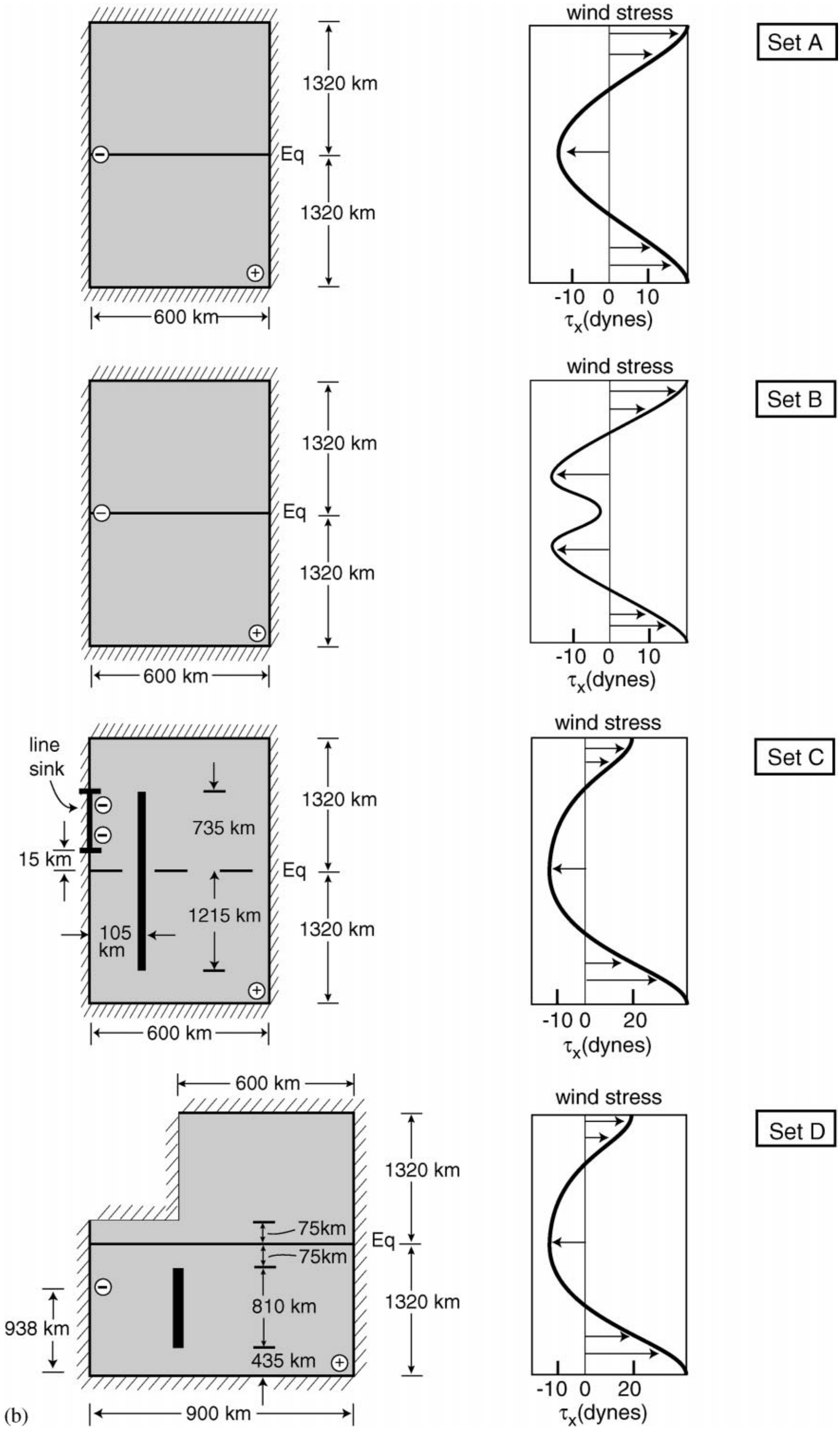

Fig. 2. (*Continued*)

1.4. Structure of presentation

Since the separation formula has not been numerically tested to the extent that the island rule has, we began our investigation by conducting a series of twenty-two (22) numerical experiments testing the validity of the formula (Sets A and B, Table 1) in an idealized basin without an island (see Fig. 2a, upper panel). The experiments involved full nonlinearity in a layer-and-a-half model of the Bleck and Boudra type. The transport was established using a specified source–sink flow that is superimposed on the wind-driven flow. We found an excellent agreement between the formula and the numerics and proceeded to a series of additional more complicated numerical experiments. These simulations involve a combination of continents and islands (Sets C and D, Table 1) where the separation formula can be directly compared to the island rule (see Fig. 2a, central and lower panels). Again, we found a very good agreement not only between the island rule and the separation formula but also between the two analytical methods and the numerics.

We shall begin our detailed presentation with a rather brief description of what both the separation formula and the island rule mean (Section 2). We then describe the numerical simulations (Section 3) and the results (Section 4). As mentioned, in all of our simulations, there was only one active layer so that the separation formula and the island rule addressed the same water mass and can be directly compared to each other. Recall that this is not the case in either our two-and-a-half layer model or the actual ocean where the separation formula corresponds to the water above the thermocline (i.e., our upper layer $\sigma_\theta \leqslant 26.2$), whereas the island rule corresponds to all the water contained between the free surface and the level of no motion (i.e., the upper and intermediate layer $\sigma_\theta \leqslant 27.5$). [It is not surprising, therefore, that the oceanic applications of the two methods do not give the same result. The separation formula (Nof, 1998) gives zero transport, whereas the island rule (Godfrey, 1989) gives 16 Sv].

2. What transports do the island rule and the separation formula measure?

Consider again the situation shown in Figs. 1a and b. An island (Australia) is surrounded by water consisting of a (stratified) upper layer whose lower interface outcrops along the $h = 0$ lines. Below this stratified layer there is a very thick homogenous layer moving very slowly (at negligible speeds); the transports in this thick layer are not necessarily small, however. The third infinitely deep layer also has negligible speeds and, as the second layer, may have non-negligible transports. (Though definitely possible, this third layer transport will not be addressed in our analysis.) The reader who is concerned about this second and third layer transport issue is reminded that the derivation of both the island rule and the separation formula involves vertical integration of the linearized equation of motion over the upper shallow layer. The original equations (prior to the vertical integration) involve pressure gradients in the deep layer below which are neglected on the ground that the velocities in the deep layer are very small. A vertical integration of these small terms over the upper layer gives a small and negligible contribution because the upper layer

Table 1

A description of the four sets of numerical experiments employing a one-and-a-half layer "reduced-gravity" version of the Bleck and Boudra model. To avoid interfacial surfacing close to the sink, a distributed sink was used in experiments 23–29 (Set C). In all experiments: $\Delta\rho/\rho = 0.00155$; $C_D = 10^{-6}$, $v = 16{,}000\ m^2\ s^{-1}$, $\Delta x, \Delta y = 15$ km, $\Delta t = 540$ s, $H = 300$ m, and $\beta = 11.5 \times 10^{-11}\ m^{-1}\ s^{-1}$. The transport was non-dimensionalized with $g'H^2/2f_0$ (which for the above values and $f_0 = 7.25 \times 10^{-5}\ s^{-1}$ gives 10 Sv)

Numerical experiment		Imposed source to sink (non-dimensional) transport	Wind field	Island	Western Boundary Currents separation from boundaries	Southern Boundary Current separation from boundaries	Shown in Figure
Set	Number of experiment						
A	1	0.00	Single maximum (see Fig. 2b)	No	Yes	Irrelevant (no island)	3
	2	0.05					3
	3	0.10					3
	4	0.15					3
	5	0.20					3
	6	0.25					3
	7	0.30					3
	8	0.35					3
	9	0.40					3, 4a, 4b
	10	0.45					3
	11	0.50					3
B	12	0.00	Double maximum (see Fig. 2b)	No	Yes	Irrelevant (no island)	5
	13	0.05					5
	14	0.10					5
	15	0.15					5
	16	0.20					5
	17	0.25					5
	18	0.30					5
	19	0.35					5
	20	0.40					5, 6a, 6b
	21	0.45					5
	22	0.50					5
C	23	0.25	Single maximum but asymmetrical (see Fig. 2b)	Yes	Yes	Yes	7
	24	0.28					7
	25	0.30					7
	26	0.33					7
	27	0.35					8a, 8b
	28	0.38					7
	29	0.40					9
D	30	0.80	Single maximum but asymmetrical (see Fig. 2b)	Yes	Yes	No	9, 10b
	31	0.90					9
	32	1.00					9

thickness is of order unity. The second layer, on the other hand, is taken to be very deep so a vertical integration of the very small terms over it gives an O(1) term. Hence, the transports are not necessarily small (see e.g., Gill and Schuman, 1979).

The separation formula (Nof, 1998) shows that the transport of all thermocline water lighter than $26.20\sigma_\theta$ (via the Indonesian passages) is given by

$$T_{\mathrm{ind}} = \frac{-1}{f_2 \rho} \int_{ABJCD} \tau' \mathrm{d}\ell + \iint_{ABCD} w \, \mathrm{d}x \, \mathrm{d}y, \tag{2.1}$$

where the first term (the combined diabatic and wind-induced transport) corresponds to an integration in a horseshoe manner [in the sense that the (open) path excludes section AED shown in Fig. 1b] and the second corresponds to an upwelling into the upper layer which must be independently computed (or specified). Here, f_2 is the Coriolis parameter along the southern western boundary current (WBC) separation latitude, T_{ind} is the transport through the Indonesian passages, w is the local upwelling through the thermocline and the remaining notation is conventional. (For clarity variables are defined in both the text and in Nomenclature.)

It should be stressed that the second integral on the right-hand side ($\iint_{ABCD} w \, \mathrm{d}x \, \mathrm{d}y$) represents the total upwelling Q and cannot be computed from the separation formula. (Note that the term "upwelling" is used here in a generic sense, i.e., it means all water converted from high to low density. Some of this conversion may occur zonally via eddy fluxes rather than through vertical motion. However, since there is no clear understanding of the eddy fluxes process yet, we lump it together with the somewhat better understood vertical motion.) All that can be computed is the first term which contains wind forcing as well as weak or moderate diabatic processes which do not need to be solved for. By "weak" and "moderate" cooling we mean here, cooling and heating that may introduce vertical and horizontal motions within the upper layer but are not severe enough to cause upwelling or downwelling through the interface bounding the upper layer from below. Namely, within the separation formula scenario there cannot be a movement of the first layer across line 1 (Fig. 1b).

To see that all "moderate" diabatic cooling or heating processes are included in the first term of (2.1), consider an upper layer with $\rho = \rho(x, y, z)$ and follow the derivation leading to (2.1) and (2.2) (in Nof, 1998) keeping the pressure terms in the general Boussinesq form of

$$\frac{1}{\rho_0} \frac{\partial P}{\partial x} \quad \text{and} \quad \frac{1}{\rho_0} \frac{\partial P}{\partial y}, \tag{2.2}$$

where ρ_0 is the density of the motionless deep layer, and P is the deviation of the hydrostatic pressure from the pressure associated with a state of rest. Upon following the derivation leading to (2.1) and (2.2) (i.e., vertically integrating the equations), defining $P^* = \int_{-\delta}^{\eta} P \, \mathrm{d}z$ (where δ is the depth of the level of no horizontal pressure gradients and η is the free surface vertical displacement) and using Leibniz's formula for the differentiation under the integral to show that $\partial P^*/\partial x = \int_{-\delta}^{\eta} (\partial P/\partial x) \, \mathrm{d}z$ (because $P = 0$ at $z = \eta, -\delta$) one sees that the right-hand side of the vertically integrated equations analogous to (2.1) and (2.2) in Nof (1998) is $(1/\rho_0)(\partial P^*/\partial x)$ and $(1/\rho_0)(\partial P^*/\partial y)$.

This implies that in the derivation of the separation formula all the pressure terms drop out even though $\rho = \rho(x, y, z)$. Consequently, (2.12) in Nof (1998) is correct even when $\rho = \rho(x, y, z)$. [Namely, statements made in Nof (1998) regarding the inability of the separation formula to handle diabatic processes are incorrect. As mentioned, the realization that (2.1) is also correct when $\rho = \rho(x, y, z)$ within the upper layer came to us after the above paper appeared.] It is also important to realize that the water immediately below the outcropping interface need not be in complete rest in the sense that *only the speeds* are required to be small; since the layer is very thick the transport can be of the same order as the upper layer transport. For an observational verification of the two-layer approach the reader is referred to Olson (1991) and Goñi et al. (1996).

Godfrey's (1989) island rule transport is given by

$$T_{\text{ind}} = \frac{1}{(f_4 - f_3)\rho} \oint_{HIBJEFGH} \tau' \mathrm{d}\ell, \tag{2.3}$$

where the (now closed) integration path *includes* the island (see Fig. 1b). The transport (2.3) is measured from the free surface all the way down to a level of no motion above the topography (say, 1400 m corresponding to $\sigma_\theta = 27.50$). The Coriolis parameters f_3 and f_4 correspond to the latitudes at the southern and northern tips of the island rather than to the WBC separation latitude (Sections 3 and 4 shown in Fig. 1b). As before, all diabatic processes are included in (2.3) provided that they are not severe enough to cause downwelling or upwelling below the level of no motion ($\sim$ 1400 m). Again, all that it takes to see this is to consider $\rho = \rho(x, y, z)$ and follow the derivation leading to (2.1) and (2.2). Upon following those steps, one notes that all the original pressure terms still cancel out even though ρ is not a constant.

In view of (2.1) and (2.3), and the observed fact that within the passages most of the throughflow corresponds to warm water[1] of $\sigma_\theta < 26.20$, we find that the upwelling from the intermediate water (green water in Fig. 1a) into the thermocline in the Pacific (yellow water in Fig. 1a) $Q(= \iint_{ABCD} w \, \mathrm{d}x \, \mathrm{d}y)$ is given by

$$Q = \frac{1}{(f_4 - f_3)\rho} \oint_{HIBJEFGH} \tau' \mathrm{d}\ell + \frac{1}{f_2 \rho} \int_{ABJCD} \tau' \mathrm{d}\ell, \tag{2.4}$$

where, as shown in Fig. 1b, $f_2 < 0, f_3 < 0, f_4 < 0$ and $|f_4| < |f_3|$.

It is important to realize that both (2.1) and (2.3) [and hence (2.4)] may contain both zonal and meridional winds. In addition, they are applicable to boundaries of any shape and form. Detailed application of (2.1) and (2.3) to the Pacific [using Hellerman and Rosenstein (1983) winds] shows that the first term of (2.1) is practically zero (Nof, 1998) whereas (2.3) gives 16 Sv (Godfrey, 1989) suggesting that the upwelling Q is 16 Sv.

In the following sections we shall apply (2.1) and (2.3) to numerical simulations (with and without islands) and examine their relationship to each other and the

[1] This means that there is no significant intermediate layer flow exiting the Pacific.

numerics. For the purpose of these comparisons, we shall consider a "reduced-gravity" layer-and-a-half model with a simplified geometry (Fig. 1b) and limit ourselves to the cases of no meridional winds.

3. Numerical simulations

To examine the validity of the separation formula (and the island rule) we consider now an application of a "reduced-gravity" Bleck and Boudra model to a rectangular basin spanning both hemispheres. As shown in Table 1 and in Figs. 2a and b, we performed four sets of experiments. First, we shall show the results of twenty-two (22) experiments (Sets A and B) whose main purpose was to examine the separation formula. Here, a source–sink flow is super-imposed on either a two-gyre or a four-gyre wind-driven flow spanning both hemispheres. (The imposed winds are symmetrical with respect to the equator.) The rationale behind these two sets of experiments (i.e., Sets A and B) is that the source along the southern boundary and the sink in the western equatorial ocean simulate the Indonesian Throughflow and force the WBC to separate at asymmetrical latitudes (enabling us to examine the separation formula).

It is important to realize in this context that, due to the linearity of the analytical problem, the source–sink flow and wind-driven flow can conceptually be solved independently from each other and then added up. The wind-driven flow is a simple two-gyre circulation whereas the source–sink flow is simply a flow along the southern and western wall. Namely, it opposes the poleward flowing WBC in the southern hemisphere implying that, in the southern hemisphere, the throughflow occurs via the interior along the eastern boundary. This is so because the Sverdrup interior transport is now greater than the WBC transport. We shall show later (Section 4.1) that this also implies that the southern separation latitude is different from that which would occur in the absence of the source–sink flow.

After completing the numerical simulation corresponding to the separation formula, we shall proceed and describe ten (10) additional experiments where the basin is the same as before except that it now contains an island that either crosses the equator (set C) or is limited to the southern hemisphere (set D). In these sets of experiments (C and D), the winds are stronger in the southern hemisphere than in the northern hemisphere, in agreement with the actual winds over the ocean. In each of these experiments, we shall vary the transport, measure the location of both WBC separations, and then plot the computed transports versus the specified transport.

3.1. The numerical model

We use a reduced gravity version of the isopynic model developed by Bleck and Boudra (1981, 1986), and later improved by Bleck and Smith (1990). The advantage of this model is the use of the "Flux-Corrected Transport" algorithm (Boris and Book, 1973; Zalesak, 1979) for the solution of the continuity equation. This algorithm employs a higher-order correction to the depth calculations and allows the layers to outcrop and stay positive definite. The resulting scheme is virtually non-diffusive and conserves mass. For these reasons, the model is the most suitable model for our problem. The active layer is an enclosed feature, while the rest of the layer has zero depth. The modeled basin is a rectangle extending from 60°S to 60°N. The mass exchange is established by withdrawing water from a sink and forcing an equal amount back into the basin via a source.

The equations of motion are the two momentum equations,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - (f_0 + \beta y)v = -g'\frac{\partial h}{\partial x} + \frac{\nu}{h}\nabla\cdot(h\nabla u) + \frac{\tau_x}{\rho h} - ku,$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - (f_0 + \beta y)u = -g'\frac{\partial h}{\partial y} + \frac{\nu}{h}\nabla\cdot(h\nabla v) + \frac{\tau_y}{\rho h} - kv$$

and the continuity equation,

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0,$$

where, as before, the notation is conventional and is given in the Nomenclature.

The model uses the Arakawa (1966) C-grid. The u-velocity points are shifted one-half grid step to the left from the h points, the v-velocity points are shifted one-half grid step down from the h points, and the vorticity points are shifted one-half grid step down from the u-velocity points. This grid allows for reducing the order of the errors in the numerical scheme. The grid scale is 15 km, and the solution is advanced in time using the leap-frog scheme with a time step of 540 s. The velocity fields are smoothed in time in order to stabilize the numerical procedure. The sources and sinks were specified by introducing a vertical velocity over a given region (i.e., the source corresponds to a mass flux of Goldsborough, 1933). All boundaries were slippery.

To make our runs more economical, we artificially magnified both β and the wind stress (by factors of five and ten, respectively), and artificially reduced both the meridional and zonal basin scales (again by a factor of five and ten, respectively). With these changes, our β was $11.5\times10^{-11}\ \mathrm{m^{-1}\,s^{-1}}$, the zonal basin size was 600 km, the meridional basin size was 1320 km (in each hemisphere), and the Coriolis parameter remained unaltered (for the new latitudes). The linear drag coefficient, k, was also increased by a factor of 10 (to $1\times10^{-6}\ \mathrm{s^{-1}}$) so that the increased wind stress is balanced with a small change in the velocity. With a grid size of 15 km, an (unaltered) reduced gravity of $1.5\times10^{-2}\ \mathrm{m\,s^{-2}}$ and an undisturbed depth of 300 m, our grid point count was dramatically reduced and our runs lasted for about 40 min (each) instead of a week or so that each run would have lasted had we not modified β, the wind stress and the basin size.

The downside of the above modifications is an unnaturally large ratio between the eddies' size (the Rossby radius) and the basin's size. As we shall shortly see, this is not really a difficulty in our runs because in our case the eddies do not play a major role in the meridional mass exchange. Furthermore, this unnatural eddy–basin-ratio issue can be easily resolved by using a very large horizontal eddy viscosity coefficient ($\nu = 16{,}000\ \mathrm{m^2\,s^{-1}}$) that eliminates any long-lived eddies such as Kuroshio rings. This essentially implies that our model is not an eddy-resolving model. It is not really an issue: because of the magnified value of β, the thickness of the Munk layer $(\nu/\beta)^{1/3}$ is still small (about 50 km) compared to the basin size.

Furthermore, the Munk layer is well resolved with our grid size of 15 km [though the inertial thickness (the Rossby radius, which is roughly 20 km) is not]. The application of the separation formula to our model is straightforward; it is discussed in detail in Nof (1998) and need not be repeated here. However, because of the surfacing interface, there is a slight difficulty in applying the island rule to our case. This is discussed below.

3.2. Island rule calculations

We shall use a slightly modified version of the classical island rule which allows for a surfacing upper layer. The extension of the original rule to this case is a bit tricky because the limit $h \to 0$ corresponds to a wind stress which must be taken to be zero even though it is actually finite. It is discussed in detail in Nof (1999) and need not be repeated here. It is sufficient to say that since, by definition, there is no flow across the outcropping line, the wind stress along it must be taken to be zero as well. This is so, even though the actual wind stress along the zero thickness line is not zero. It is rationalized by the argument that in the derivation of the rule one assumes that the stress along the bottom of the upper layer is zero. When $h \to 0$, this assumption is no longer valid because, in this limit, the bottom stress must cancel the surface stress. In other words, in the numerical model, it is the linear drag terms (and not the lateral friction) that balance the wind stress in the same manner that the linear drag balances the pressure drop along a western boundary current. This is not an entirely satisfactory scenario as it begs the question of how close to the zero thickness line one can get and still take the bottom stress to be zero. At this stage, there is no satisfactory answer to this question, and we have no choice but to accept this difficulty as a possible weakness of the model.

Before describing our one-and-a-half-layer model results, it is appropriate to point out that we chose to introduce the throughflow as a source–sink flow beginning in the southeast corner and ending in the western equatorial ocean even though the actual throughflow probably enters the Pacific in the southwest corner rather than the southeast. This chosen transport path in our one-and-a-half layer model is deliberately unrealistic, as our model does not contain intermediate water which can handle water entering from the southwest.

4. Numerical results

Our results are shown in Figs. 3–10, where the analytical predictions and their numerical counterparts are plotted. We shall discuss our results for each set separately.

4.1. Set A

Fig. 3 shows the relationship between the transport predicted by the separation formula (using the numerically observed WBC separation latitudes) and the actual specified transport for the first 11 experiments (see Table 1). Note that the source–sink flow does not appear explicitly in (2.1) or (2.4). It is, nevertheless, implicitly included via the position of the WBC separation which adjusts to the transport. We see from Fig. 3 that even though the numerical model is based on the primitive equations whereas the analytical model is quasi-linear [in the sense that the advection terms are neglected but the interface displacements are of O(1)], the agreement is excellent. Typical transport and thickness contours are shown in Fig. 4. We see that the imposed sink–source transport crosses the south Pacific first along the eastern boundary and then along the equator.

The asymmetry between the thickness contours along the northern and southern boundaries (seen in Fig. 4) can be explained by examining the linearized, vertically integrated momentum equation in the x-direction. The equation is written as

$$-fV = \frac{g'}{2}\frac{\partial}{\partial x}(h^2) + \frac{\tau}{\rho} \tag{4.1}$$

(where V is the vertically integrated transport), and is integrated from the position of the separated flow ($h = 0$; at $x = \xi$) to the eastern boundary ($x = L$; $h = H_e$) to give

$$fT = g'H_e^2/2 - \int_{\xi}^{L} \frac{\tau}{\rho}\,\mathrm{d}x, \tag{4.2}$$

where T is the total meridional transport across the basin.

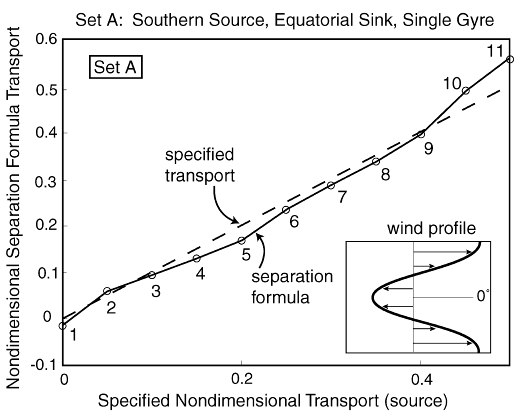

Fig. 3. A comparison between the transport predicted by the separation formula from the (numerical) WBC separation latitude and the wind field (solid line) and the specified transport (dashed line) in a rectangular ocean with a source in the southeast corner and a sink in the western equatorial region. The 11 experiments shown here correspond to Set A (see Table 1). In addition to the source–sink flow, the basin is subject to a wind field with a maximum along the equator (see inset). Note that the agreement between the two is excellent. The transport is non-dimensionalized by $g'H^2/2f_0$, where $g' = 2\times10^{-2}$ m s^{-2}; $H = 300$ m; and $f_0 = 9\times10^{-5}$ s^{-1}.

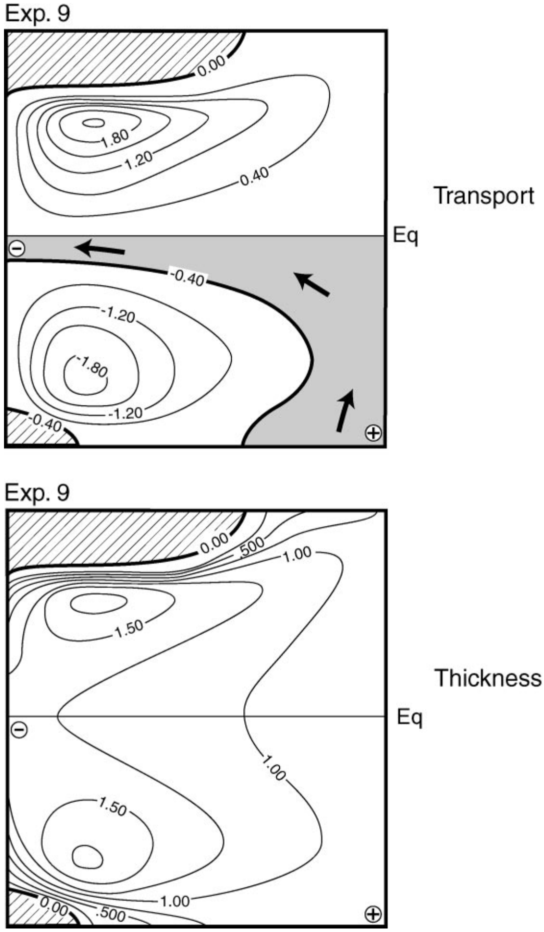

Fig. 4. Typical (non-dimensional) transport and thickness contours for Set A (Exp. 9). The net sink–source transport is shaded. Hatched regions denote areas with no upper layer (thickness non-dimensionalized into the undisturbed depth of 300 m). Note that there is a 1 : 4 ratio between the throughflow mass flux (0.4) and the net amount circulating forever in the South Atlantic (1.6). Also, note that the source is a point source even though it appears to be a line source. This false impression is due to the absence of transport contours of less than 0.40 (which are connected directly to the point source).

Since there are no meridional winds in these examples, the thickness does not vary along the eastern boundary and (4.2) is applicable to both boundaries with the same H_e. We now note that along the southern boundary $T > 0$ and $f < 0$ whereas along the northern boundary $T = 0$. From this it immediately follows that, for symmetrical winds, $\xi_{north} > \xi_{south}$ because along the southern boundary the stress term must be integrated over a longer distance to balance the pressure term and the mass transport term. It implies that along the northern boundary the separation line is situated farther to the east than the separation line along the southern boundary.

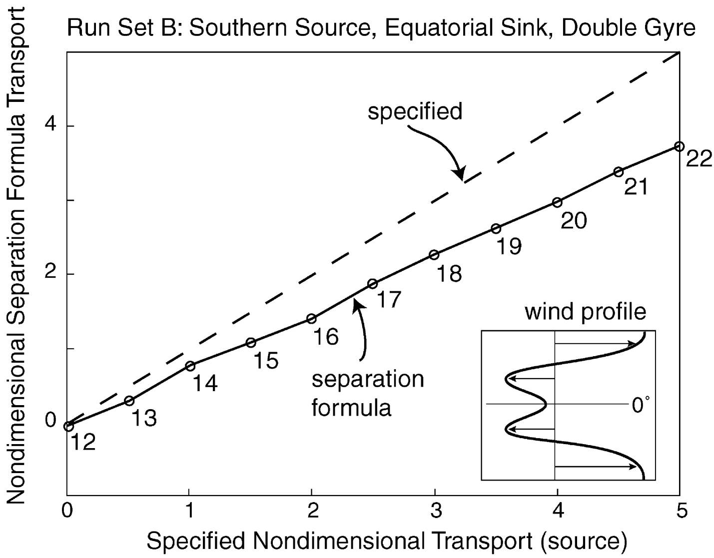

Fig. 5. A comparison between the transport predicted by the separation formula (solid line) and the specified transport (dashed line) in a rectangular ocean (with a source in the southeast corner and an equatorial sink) subject to a wind profile with *two* maximums in low latitudes (Set B).

4.2. Set B

Fig. 5 shows the relationship between the transport given by the separation formula and the actual specified transport for the next 10 numerical experiments. Note that the only difference between this set and the earlier one is that, here, the wind profile has an off-equatorial maximum resulting in a two-gyre (instead of one gyre) circulation in each hemisphere. We see that the agreement between the two is still very good though it is not as good as in Set A. This worsening in the agreement is due to the strong zonal jet created between the two gyres (see the imposed transport path in Fig. 6). We have calculated the size of the various terms and concluded that, along the zonal jets the zonal frictional forces (neglected in the formula) become important and can no longer be ignored.

4.3. Set C

Fig. 7 shows a comparison between the island rule prediction for the transport, the separation formula prediction, and the actual specified transport. Recall that, in this set of experiments, the island is spanning across the equator. The separation formula agrees with the specified transport a bit better than the island rule calculations because a smaller fraction of the formula's integration path cuts through the zonal jets (where friction which is neglected in both the formula and the rule cannot be ignored). This can be clearly seen in Figs. 8a and b which show the transport and thickness

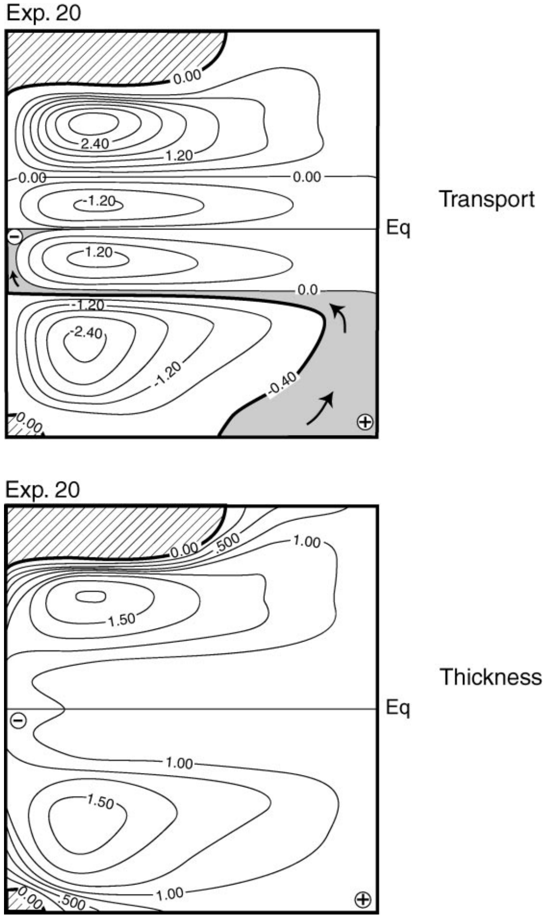

Fig. 6. Typical (non-dimensional) transport and thickness contours for Set B (Exp. 20).

contours. Note that in this set of experiments the imposed mass flux was adjusted in such a way that a separation from the island occurs. This is not always the case and a stronger mass flux forces some of the fluid to circulate forever around the island (Set D).

4.4. Set D

In this set of experiments, the southern WBC separates from the basin's boundary rather than from the island (see Fig. 2) so that a direct comparison between the rule and the formula cannot be made. This is because the island rule measures the total transport circulating around the island whereas the separation formula measures only the net transport between the source and the sink. In our case (see Fig. 10a), there is water circulating forever around the island (without ever entering the sink) which is

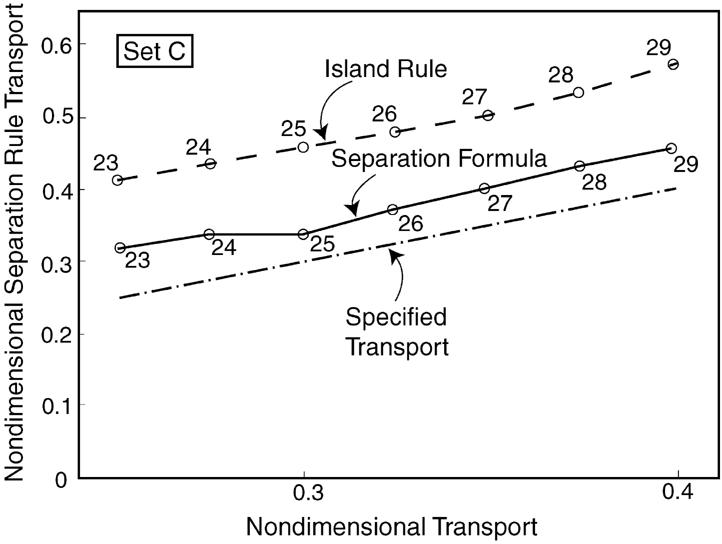

Fig. 7. An intercomparison of the non-dimensional transport predicted by the separation formula (solid line), the island rule (dashed line) and the specified source–sink transport (dashed-dotted line) for a basin with an island spanning the equator (see middle panel in Fig. 2a). The seven simulations shown correspond to Set C. The flow is driven by a point source in the southeast corner, a distributed sink (to prevent the basin from "drying up") in the northwest and a zonal wind profile with a maximum along the equator.

measured by the island rule but is not measured by the separation formula. To overcome this, a comparison is made between the specified transport, the separation formula transport, and the difference between the island rule transport and the numerically measured transport circulating forever around the island. This comparison is shown in Fig. 9. Typical transport and thickness contours are shown in Fig. 10. As in the previous sets, there is a slightly better (though not significantly better) agreement between the separation formula and the actual specified transport than between the island rule and the numerics. As before, this is due to the fact that a smaller fraction of the separation formula integration path cuts through the zonal jets where the assumptions of no zonal friction and small nonlinearity are violated.

5. Summary and discussion

We showed that it is possible to compute the total upwelling into the thermocline in the Pacific ($\sigma_\theta = 26.20$) from the difference between the transport given by the separation formula and the transport given by the island rule (provided that friction in the Indonesian Seas is ignored, and that, as the observations suggest, most of the throughflow occurs in the upper layer). To show this, we constructed a two-and-a-half layer model to which both the separation formula and the island rule apply. Direct numerical verification of the two-and-a-half-layer model is very difficult (due to the

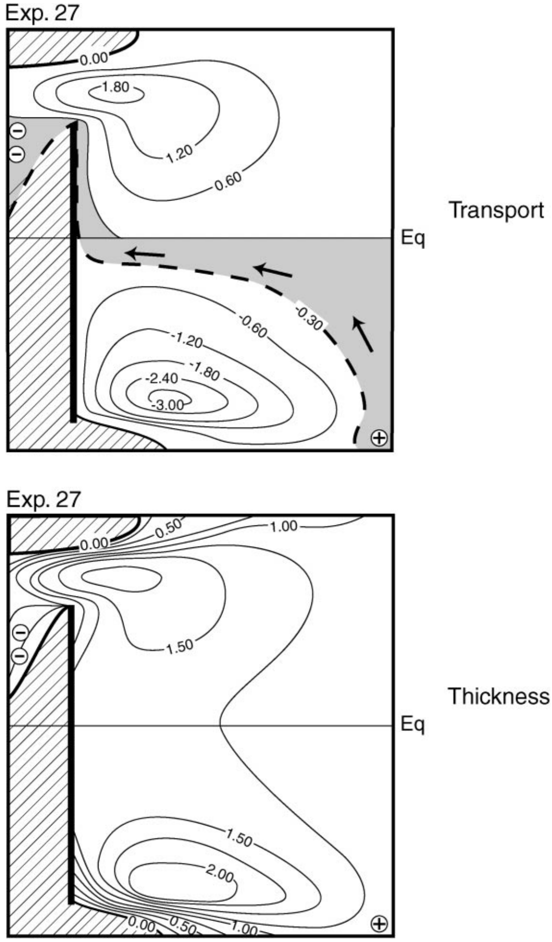

Fig. 8. Typical (non-dimensional) transport and thickness contours for a flow driven by a point source in the southeast corner and a distributed sink in the northwest superimposed on a field driven by zonal winds with a maximum along the equator. These contours correspond to Exp. 27, Set C; the basin contains a meridional island spanning across the equator; the western boundary currents separate from the boundary in the northern hemisphere and from the island in the southern hemisphere. Regions which do not contain an upper layer are hatched. Shaded regions denote the throughflow.

required parameterization of vertical mixing) but we did verify the validity of the separation formula and its consistency with the island rule in a layer-and-a-half model. Specifically, we performed four sets of numerical simulations (Table 1). First, we performed twenty-two (22) experiments to examine the validity of the separation formula (Sets A and B, Fig. 2). All showed an excellent agreement between the formula and the numerics (Figs. 3–6) suggesting that the formula's predictions are reasonable. Second, we performed ten (10) more simulations (Sets C and D, Fig. 2) which included

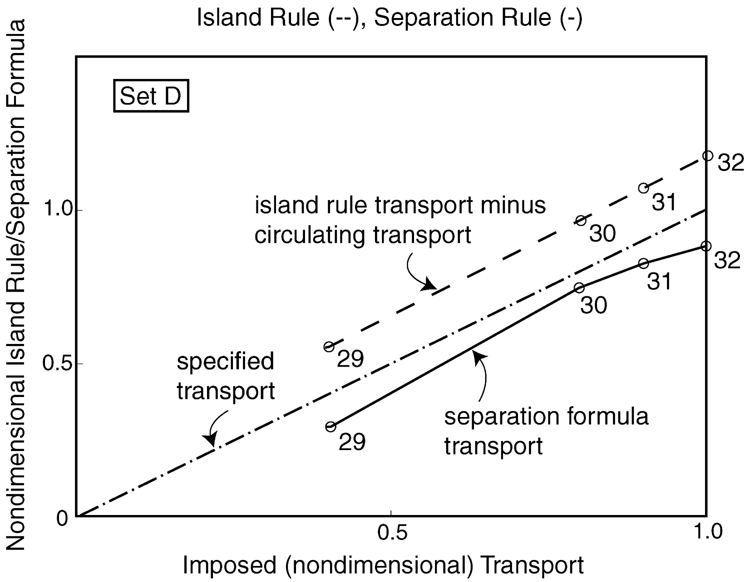

Fig. 9. The same as in Fig. 7 except that the shown results are for Set D rather than C. Here, the southern hemisphere separation takes place from the boundary and not the island (see Fig. 2, lower panel) so that in order to compare the island rule calculation to the separation formula it is necessary to add the (numerically measured) transport between the island and the western boundary.

the island rule and the separation formula simultaneously. Again, very good general agreement has been found (Figs. 7–10) suggesting that the two methods are consistent and compatible with each other. The agreement between the island rule and Set C is not so good, however, primarily due to zonal jets cutting through the integration area (see for example Fig. 8, southern tip of island). Zonal jets are subject to significant linear drag which is neglected in the analytics.

According to our two-and-a-half layer analytical model, the difference between the two predictions for the Pacific is the upwelling (captured by the Island rule but not by the separation formula). Note that on their own neither of the two methods could have predicted the upwelled amount. The island rule gives the amount that ultimately ends up in the Indonesian Throughflow (as the observed warm water is lighter than $26.20\sigma_\theta$) but it does not tell us where the water is coming from. In fact, a simple (and an erroneous) interpretation of the island rule would have one think that it is Pacific water of the same density as the water in the passages (i.e. surface Pacific water of $26.20\sigma_\theta$) that is directly forced into the Indonesian passages. The separation formula, on the other hand, tells us that it is not simply surface water that circulates around Australia and enters the passages. It says that all water that ends up as surface water in the Indonesian passages ($\sigma_\theta < 26.20$) must upwell into the thermocline throughout the Pacific. The separation formula does not say, however, how much is upwelled. It is the combination of the two methods and the conceptual construction of the two-and-a-half layer model that allows us to compute the upwelled amount.

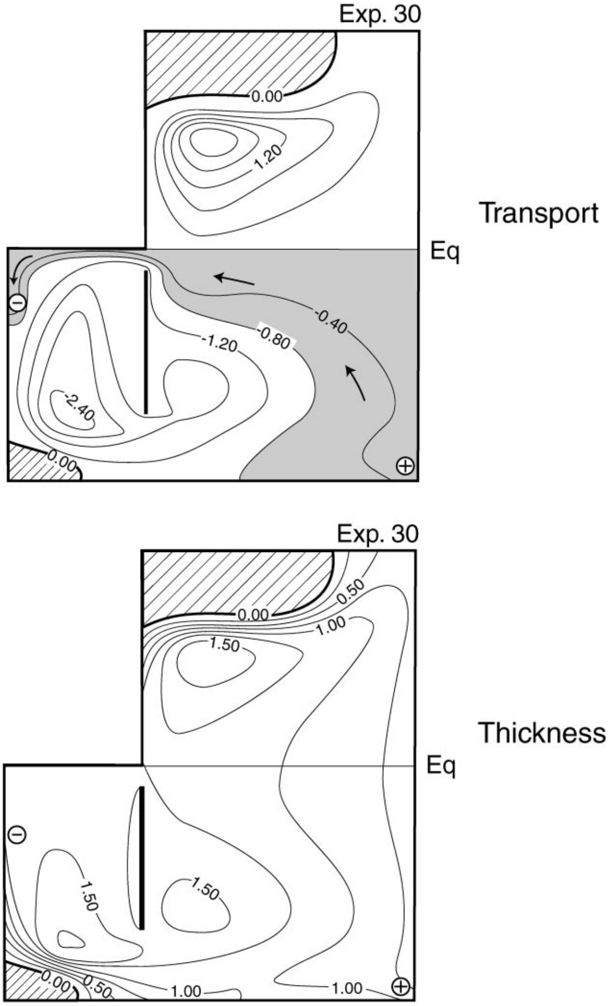

Fig. 10. Typical (non-dimensional) transport and thickness contours for set D (the shown results are of experiment 30).

It should be pointed out that vertical recirculation cells (going through either cross section ① or ②) resulting from a combined upwelling and downwelling are neglected in our scenario. To see this, we note that water of less than $26.20\sigma_\theta$ can go north past DC (Fig. 1b) and subduct. It can then return below as water with $\sigma_\theta > 26.20$. However, since we take the throughflow to consist of water with $\sigma_\theta < 26.20$, this subducted water must be ultimately upwelled back into the thermocline forming a closed vertical recirculation cell (Fig. 11). (Incidentally, for this case with upwelling the separation formula states that an amount equal to that which goes northward

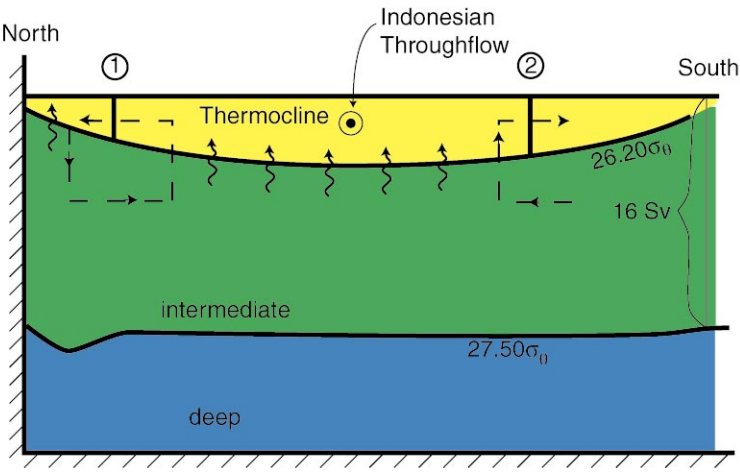

Fig. 11. A schematic diagram of the upwelling scenario. Sections ①and ② correspond to those shown in Fig. 1b. Our upwelling transport (calculated as the difference between the island rule and the separation formula) is shown with the "wiggly" solid arrows. Recirculation cells (broken lines) which go through either cross section ①or ② are neglected in our analysis.

through DC must go southward through AB but this is not important for the present discussion.) Such vertical recirculation cells superimposed on our upwelling pattern are certainly possible but, in line with common practice which routinely ignores recirculation cells (e.g., general circulation models, jet interaction problems), we neglect them. It is difficult to estimate the magnitude of this recirculation but for 24°N the analysis by Bryden et al. (1991) implies a southward flow of about 5 Sv immediately below $26.20\sigma_\theta$. This should be the maximum value for the recirculation as there is no evidence that this water ever upwells back into the thermocline. At this stage of our knowledge, this is all that can be said about the recirculation cells.

An alternative to the Pacific upwelling scenario is that the combined formula–island two-and-a-half layer model is flawed. Two aspects of the model could perhaps be problematic. The first is the fact that friction in the Indonesian passages is absent from the island rule calculation. This possibility is very realistic (see e.g., Pedlosky et al., 1998) but all that friction can do (in most cases) is reduce the estimated upwelled mount. This possibility is, therefore, rejected as a process that can fundamentally change our conclusions (though it can definitely change the amount of upwelled water). The second possibility is that most of the Indonesian Throughflow occurs below the $26.20\sigma_\theta$. This possibility was already touched upon in Nof (1998) and is rejected on the ground that, although there is definitely a throughflow of deep water, much of the throughflow is lighter than $26.20\sigma_\theta$. Nevertheless, it will be useful if the next step of the numerical investigation will involve two active layers completely surrounding the island between the layers so that the effect of diapycnal exchange can be examined in detail.

Acknowledgements

We wish to thank S. Godfrey for many very useful discussions and Mike Spall and Chris Hughes for carefully reviewing the article. This study was supported by the National Science Foundation (NSF) under grants OCE 9102025 and OCE 9503816, National Aeronautics and Space Administration (NASA) grants NAGW-4883 and NAG5-4813, Office of Naval Research (ONR) grant N00014-96-1-0541, and Binational Science Foundation grant 96-105.

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