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Lenses generated by intermittent currents

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Abstract—A nonlinear mechanism for the generation of anticyclonic lens-like eddies from boundary currents is proposed. In contrast to the familiar generation processes that rely on unstable long waves that grow and close upon themselves or vortex shedding due to the geometry of the boundary, the present mechanism is related to intermittency in the current's mass transport. The essence of the new mechanism is that intermittencies in the transport (such as those in the Denmark Strait or the Mediterranean outflow) lead to unbalanced patches of fluid which break up into a discrete set of eddies that interact with the boundary.

The process is highly nonlinear because both the amplitude and the Rossby number are of order unity. It is modeled as follows: we begin with a rectangular box containing the motionless (light) fluid near the boundary. At, say, $t=0$, the conceptual box is removed and the unbalanced fluid undergoes two main processes. The first involves the establishment of a set of eddies via breakup and geostrophic adjustment, whereas the second is associated with the interaction of the set with the wall. These two processes are examined independently even though in reality the processes are, obviously, taking place at the same time.

To examine the first process we consider the nonlinear collapse of a (light) rectangular box in the open ocean away from the boundary. The breakup process involves, of course, some sort of instability (because the patch does not remain intact) but this is not necessarily related to the long wave instability that is usually associated with long gravity currents. The general structure of the resulting final chain of eddies can be computed analytically by using the usual connecting principles, the conservation of potential vorticity and mass. It turns out, however, that the number of eddies and their detailed structure cannot be computed unless one invokes an additional constraint. To resolve this closure difficulty, the integrated angular momentum constraint, which is rarely used in oceanographic modeling, has been applied.

The details of the second process (i.e. chain–wall interaction) are examined with the aid of the (above) results for the chained offshore eddies and the single eddy–wall interaction analysis of NOF (1988, *Journal of Marine Research*, **46**, 527–555). A combination of these two studies shows that a chain of lens-like eddies forced against a wall would leak fluid until all the eddies in the chain are merely 'kissing' the wall.

Simple qualitative laboratory experiments on a rotating table support the conclusion that intermittencies in the current's transport lead to a group of eddies that leak along the wall. Such a group is formed even if the equivalent steady current is *stable* to long wave perturbation, i.e. the laboratory current would not have broken up had it not been terminating shortly after its formation. It is suggested that the observed mid-Atlantic eddies resulting from the Mediterranean outflow (Meddies) might be formed by a mechanism similar to our newly suggested process.

1. INTRODUCTION

ANTICYCLONIC eddies having a lens-like structure are common in all parts of the ocean. Examples are the warm-core rings north of the Gulf Stream (RING GROUP, 1981), the so-

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called 'Meddies' (McDowell and Rossby, 1978), Amazonian eddies (e.g. Nof, 1981), and numerous mid-depth submesoscale vortices (McWilliams, 1985). In this paper we shall propose a new generation mechanism for lenses that originate from outflows such as the Mediterranean, the Denmark Straits or the Amazon.

So far, it has been believed that lenses are generated by (i) the instability of boundary currents to long wave perturbation (e.g. Stern, 1980; Griffiths and Linden, 1981, 1982; Stern *et al.*, 1982; Griffiths and Hopfinger, 1983), (ii) abrupt changes in the geometry of the boundary along which the current is flowing (D'Asaro, 1988a,b), and (iii) mixing on continental shelves (McWilliams, 1985). While all of these mechanisms are certainly plausible, it is shown in what follows that a fourth generation mechanism might also be active in many outflows. This fourth mechanism is related to intermittencies in the outflow's transport which, as it turns out, causes the establishment of a discrete set of lenses.

The reader is warned in advance that, at first glance, it may appear that there is little difference between the proposed intermittency processes and the long wave instability of steady currents because both mechanisms produce eddies from coastal flows. However, a closer look will show that there is an important difference between the two. While the steady current mechanism relies on the current being vigorously unstable to long wave perturbation, the intermittent process produces eddies even if the coastal current is *stable* to long wave perturbation (e.g. the stable current considered by Paldor, 1983). The reader may find it helpful to look at the problem at hand as being to some degree analogous to eddies generated from a stream of smoke. While a *steady* stream may produce small eddies if it is unstable to small long wave perturbation, *puffs* of smoke produce smoke rings which are conceptually analogous to the eddies produced by intermittency.*

Background

Strong fluctuations and pulsations in the transport of the Mediterranean outflow, the outflow from the Denmark Straits and the Amazon have been observed many times (Worthington, 1969; Gibbs, 1970; Ross, 1976; Candela *et al.*, 1989). Such pulses lead to unbalanced patches of water that must break up into a group of lenses regardless of the stability properties of their equivalent steady flows. To examine these processes in detail we shall simplify the problem to that of a single pulse (Fig. 1). The pulse can be a result of various processes such as high atmospheric pressure associated with storms above the Mediterranean (e.g. Garrett, 1983; Garrett and Majaess, 1984; Candela *et al.*, 1989) or heavy rains in the case of the Amazon.

The immediate result of such bursts is the establishment of a short coastal current that persists only for the duration of the pulse. The current is bounded by a front ($h = 0$) on one side and a wall on the other; its 'head' is associated with a 'nose' that travels in a similar fashion to a Kelvin wave (Fig. 2). Namely, the head of the current is pushed forward by the fluid advancing behind it. Once the source is turned off no new fluid is added to the current and, consequently, it is no longer in balance. Some sort of adjustment must take place and we shall focus on this particular process. It should be pointed out that there are actually *three* adjustment processes in the above scenario. The first is associated with the

*Note added in proof: very recently, after the acceptance of this paper, the author became aware of the independent study of Bliakov and Volkov (1980) who have qualitatively proposed a similar mechanism.

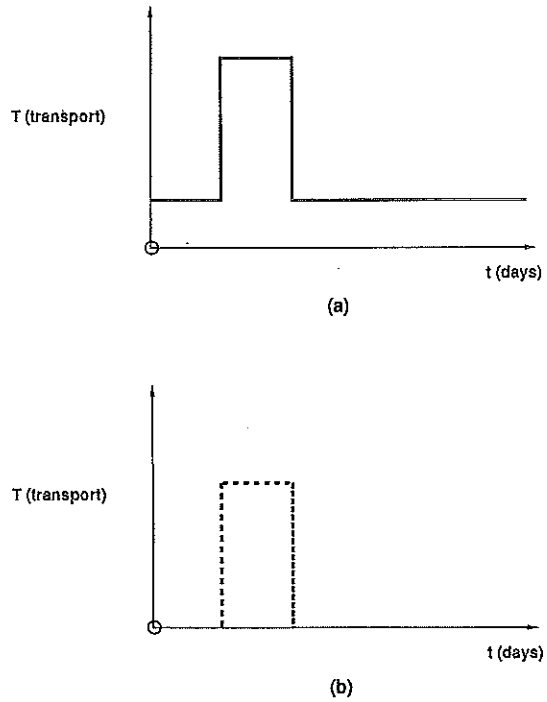

Fig. 1. Schematic diagram of the assumed transport pulse (a) and the simplified burst (b) which ignores the small steady flux. The situation described in (b) will be used throughout this study.

establishment of the coastal current which is geostrophic in the cross-stream direction, the second is related to the breakup processes (and the formation of eddies), and the third is the establishment of the leakages along the wall. We will pay almost no attention to the first adjustment and little to the third. The second adjustment process will attract most of our attention (hereafter, this second adjustment will be simply referred to as 'adjustment').

Formulation

As an idealization of the above scenario, the burst current is replaced by a box adjacent to the shoreline (Fig. 3). The box contains the light fluid which is restrained from moving because the pressure gradients are exerted on the solid walls. Once the box is removed, however, our so-called second adjustment process takes place. The patch of fluid collapses and undergoes a process similar to that which a current that is turned off would go through. We shall see that such a collapse process leads to a breakup of the patch and the formation of a discrete set of eddies that interact with the wall. Despite our simplifications, this highly nonlinear process is still quite complicated and, for mathematical tractability, further simplifications are necessary.

In view of this, we first look at the collapse of the box away from the shoreline (Fig. 4) and then bring the box back to the coast. The first step will include, then, the collapse of a box in the open ocean (the second adjustment) and the second will involve the interaction

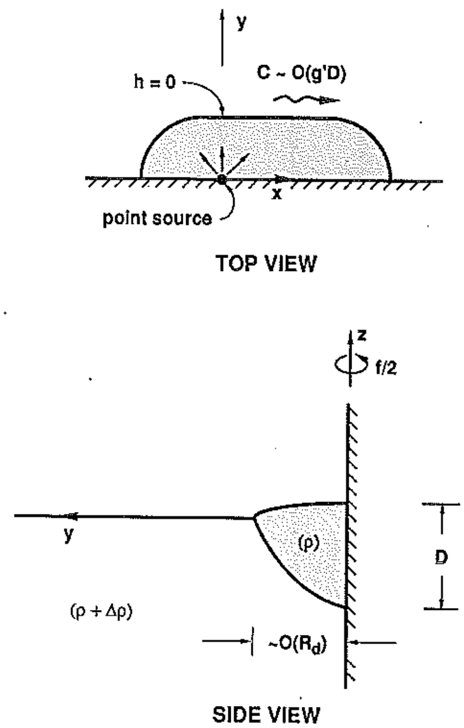

Fig. 2. A schematic diagram of a coastal current resulting from a burst of light water. The flow hugs the coast on the right-hand side and its nose propagates at a speed comparable to the gravity wave speed $(g'D)^{1/2}$ (where g' is the 'reduced gravity' $g\Delta\rho/\rho$ and D is the near wall depth). To simplify our analysis it will be assumed that this initial configuration can be idealized by a box that contained the relatively light fluid (Fig. 3).

of the resulting features with the wall (the third adjustment). Note that, as mentioned, all the processes in question are highly nonlinear because both the amplitude and the Rossby number are of order unity.

As mentioned, much of the discussion in the paper is concerned with the offshore collapse problem (i.e. the second adjustment, Section 2). The process is solved by connecting the initial and final state without solving for the time-dependent process. In contrast to most adjustment problems which can be solved by using conservation of potential vorticity and mass, the problem at hand cannot be closed unless one uses an additional constraint because the number of lenses generated by the breakup is not known *a priori*. It turns out that application of the conservation of integrated angular momentum—which is not frequently used in oceanography—enables one to close the problem without solving for the time-dependent process. Two solutions are presented in detail; the first is a perturbation expansion for boxes whose dimensions are small compared to the deformation radius and the second is a mixed (analytical–numerical) solution for boxes whose dimensions are comparable to the deformation radius. These solutions give the sizes and geometrical arrangement of the final lenses as well as the associated energy losses.

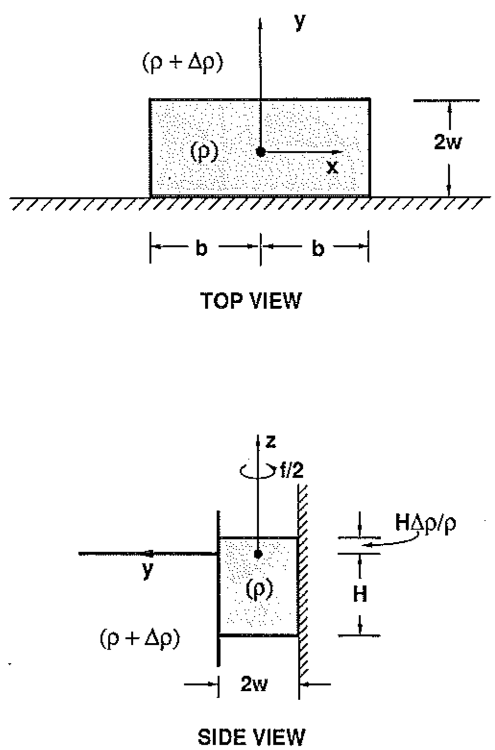

Fig. 3. Schematic diagram of the initial conditions for the model under study. The light fluid (shaded) is contained within a rectangular box that is later removed. After withdrawal the light fluid undergoes a collapse and an adjustment process that involves the generation of eddies and their interaction with the wall. Note that, from a mathematical point of view, the problem is identical to that of a collapse sandwiched between two infinitely deep layers. Such a situation corresponds to the Mediterranean outflow.

After presenting the above derivations, the offshore solutions are 'brought back to the shore' and the interaction of the eddy chain with the wall (i.e. the third adjustment) is examined. With the aid of the Nof (1988) results for a single eddy interacting with a wall, the interaction of the chain of eddies is examined (Section 3). It is shown that, as a result of the wall, each of the eddies in the chain leaks fluid to the right (looking offshore in the northern hemisphere) until the eddies are merely 'kissing' the wall.

A simple qualitative laboratory experiment on a rotating table was performed in order to examine the processes suggested by the mathematical solution (Section 4). A coastal current was generated by injecting dyed fresh water along the wall of a salt water container. At a desired time the injection was abruptly terminated and the resulting events were photographed from above. As expected, within several rotation periods the current broke up into a chain of eddies that interacted with the wall. Weak leakages along the wall were noted; such flows are in agreement with the theoretical predictions.

Finally, a qualitative application of the theory and experiment to the Mediterranean outflow and the formation of Meddies is discussed in some detail (Section 5). (Note that parts of this analysis as well as some of the theoretical aspects were rather briefly reported in Nof, 1990c.)

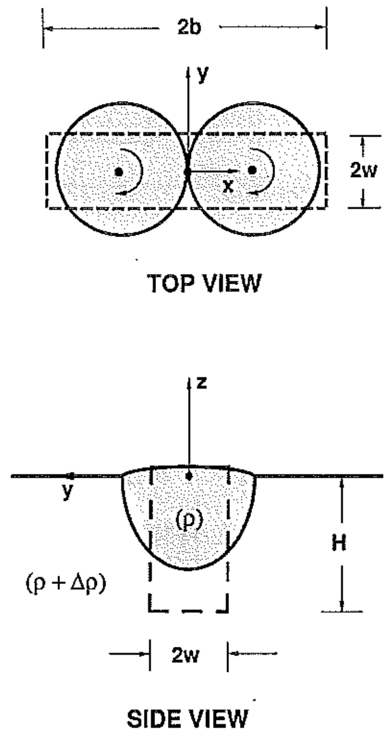

Fig. 4. A schematic diagram of our collapse process (i.e. the second adjustment). In contrast to Fig. 3, where the collapse is taking place in the vicinity of the wall, we consider here the collapse of a rectangular box (dashed line) in the open ocean away from boundaries. The collapse produces a group of eddies whose detailed structure is computed.

2. THE OFFSHORE COLLAPSE PROBLEM

Consider again the rectangular box shown by the dashed line in Fig. 4. Upon the removal of the box the light fluid (shaded) undergoes an adjustment process and breaks up into a group of steady eddies. Our aim is to find the eddies' structure resulting from a *given* box. It turns out, however, that, from a mathematical point of view, it is much simpler to solve the inverse problem. Namely, we shall take various groups of identical eddies and look for the corresponding initial boxes.

General constraints

In addition to the conservation of potential vorticity (f/H), the following constraints connect the initial and final states.

Conservation of volume.

$$2wbH = n\pi \int_0^R hr \, \mathrm{d}r, \tag{2.1}$$

where $2w$, $2b$ and H are the width, length and depth of the original box, R is the radius of the (final) eddies, h is their depth, and n is their number.

Conservation of momentum and the center of mass. It is a simple matter to show using the shallow water equations that, for a patch whose depth vanishes along the rim (BALL, 1963; NOF, 1990b) the following integrals must be satisfied,

$$\frac{d}{dt}\int\int h(u - fy)\,dx\,dy = \frac{d}{dt}\int\int h(v + fz)\,dx\,dy = 0. \qquad (2.2)$$

Here, the (Cartesian) notation is conventional (i.e. h is the depth, u and v are the speeds and f is the Coriolis parameter) and the integrals are taken over the whole patch. Since in both the (steady) initial and final states the streamfunction ψ [defined by $\partial\psi/\partial x = vh$; $\partial\psi/\partial y = -uh$] is a constant along the edge, it follows from (2.2) that,

$$\frac{d}{dt}\int\int hy\,dx\,dy = \frac{d}{dt}\int\int hx\,dx\,dy = 0,$$

which implies that the final eddies' center of gravity must coincide with the initial center of mass.

Torque. The conservation of integrated angular momentum for a patch whose depth vanishes along the rim is discussed by BALL (1963), CUSHMAN-ROISIN (1989) and NOF (1990a,b). In polar coordinates (r, θ), it is given by

$$\frac{d}{dt}\int\int (fr^2/2 + rv_\theta)hr\,dr\,d\theta = 0, \qquad (2.3)$$

where, as before, the integration is done over the whole area and v_θ is the orbital velocity. Application of (2.3) to the problem at hand gives

$$\frac{fH}{3}(w^3b + b^3w) = \pi\sum_{k=1}^{n}\int_0^R\left[\frac{f}{2}(d_k^2 + r^2) + rv_\theta\right]hr\,dr, \qquad (2.4)$$

where d_k is the distance of the kth eddy from the origin (i.e. the center of mass which remains unaltered) and it has been taken into account that the integrated angular momentum of a vortex situated a distance 'd' away from the origin includes the term $fd^2/2$ (e.g. NOF, 1990a).

The term on the right-hand side of (2.4) represents half of the final angular momentum ($A_f/2$). For an odd number of eddies it can be expressed as

$$\frac{A_f}{2}\pi\left\{\underbrace{\int_0^R (fr^2/2 + rv_\theta)hr\,dr}_{\text{angular momentum of central eddy}} + \underbrace{2\sum_{k=1}^{(n-1)/2}\int_0^R\left\{\frac{f}{2}[(2kR)^2 + r^2] + rv_\theta\right\}hr\,dr}_{\text{angular momentum of } (n-1)/2 \text{ lenses on each side of the central eddy}}\right\} \qquad (2.4a)$$

and for an even number,

$$\frac{A_f}{2} = 2\pi\underbrace{\sum_{k=1}^{n/2}\int_0^R\left\{\frac{f}{2}[(2k-1)^2R^2 + r^2] + rv_\theta\right\}hr\,dr}_{\text{angular momentum of } n/2 \text{ lenses on each side of center}}. \qquad (2.4b)$$

With the aid of the above constraints the solution to the offshore collapse problem will be derived in the next subsection.

Analytical solution for the collapse of a deep and narrow box

In this subsection we focus on eddies resulting from boxes whose horizontal dimensions are small compared to the deformation radius, $(g'H)^{1/2}/f$, because it turns out that such boxes produce eddies that have a potential vorticity that is nearly zero so that they can be solved analytically. The equations that govern each individual eddy are the momentum and potential vorticity equations which (in polar coordinates) take the form,

$$v_\theta^2/r + fv_\theta = g'\,\mathrm{d}h/\mathrm{d}r \tag{2.5a}$$

$$\mathrm{d}v_\theta/\mathrm{d}r + v_\theta/r + f = hf/H. \tag{2.5b}$$

The scales that we focus on are given by the following nondimensional variables,

$$\left.\begin{array}{lll} \varepsilon = (b/R_\mathrm{d})^2 \ll 1; & w^* = w/\varepsilon^{3/2}R_\mathrm{d}; & h^* = h/\varepsilon H \\ r^* = r/\varepsilon^{1/2}R_\mathrm{d}; & v_\theta^* = v_\theta/\varepsilon^{1/2}(g'H)^{1/2}; & R^* = R/\varepsilon^{1/2}R_\mathrm{d} \\ r_\mathrm{d} = (g'H)^{1/2}/f. & & \end{array}\right\} \tag{2.6}$$

Here, R^* is the final radius of each eddy, and the number of eddies involved is of $O(1)$. Recall that, for convenience, we assume that the size of the box is not entirely known in advance. Only the length of the box, $2b$, is taken to be known *a priori*. Hence, for a given number of identical chained eddies n, there are now two unknowns—the radius of each final vortex and the width of the initial box.

Using (2.6) we find that the nondimensional form of (2.5) is

$$(v_\theta)^{2*}/r^* + v_\theta^* = \mathrm{d}h^*/\mathrm{d}r^* \tag{2.7a}$$

$$\mathrm{d}v_\theta^*/\mathrm{d}r^* + v_\theta^*/r^* = \varepsilon h^*. \tag{2.7b}$$

By expanding h^* and v_θ^* in a power series of ε (e.g. $h^* = h^{(0)} + \varepsilon h^{(1)} + \ldots$ $v_\theta^* = v_\theta^{(0)} + \varepsilon v_\theta^{(1)} + \ldots$), considering the conditions that $v_\theta^* = 0$ at $r^* = 0$ and that $h^* = 0$ at $r^* = R^*$, the following zeroth-order solution is obtained

$$v_\theta^{(0)} = -r^*/2; \qquad h^{(0)} = [(R^*)^2 - (r^*)^2]/8. \tag{2.8}$$

The nondimensional forms for the conservation of volume and angular momentum are

$$w^* = \pi n \int_0^R h^* r^*\,\mathrm{d}r^* \tag{2.9}$$

$$\frac{1}{3\pi}[w^* + \varepsilon^2(w^*)^3] = \int_0^{R^*} [(r^*)^2/2 + r^* v_\theta^*] h^* r^*\,\mathrm{d}r^* + \sum_{k=1}^{(n-1)/2} \int_0^{R^*} \left\{\frac{1}{2}[4k^2(R^*)^2 + (r^*)^2] + r^* v_\theta^*\right\} h^* r^*\,\mathrm{d}r^* \tag{2.10a}$$

for n-odd or

$$\frac{1}{6\pi}[w^* + \varepsilon^2(w^*)^3] = \sum_{k=1}^{n/2}\left\{\frac{1}{2}[(2k-1)^2(R^*)^2 + (r^*)^2] + r^* v_\theta^*\right\} h^* r^* \, dr^* \quad (2.10b)$$

for n-even. Substitution of our expansions into (2.9)–(2.10) gives to leading order,

$$w^{(0)} = \pi n (R^{(0)})^4/64 \quad (2.11)$$

and

$$\frac{w^{(0)}}{3\pi} = \begin{cases} \dfrac{1}{2}\displaystyle\sum_{k=1}^{(n-1)/2}\int_0^{R^{(0)}} k^2 (R^{(0)})^2[(R^{(0)})^2 - (r^*)^2] r^* \, dr^* & n\text{-odd} \\ \dfrac{1}{4}\displaystyle\sum_{k=1}^{n/2}\int_0^{R^{(0)}} \dfrac{(2k-1)^2}{2}(r^{(0)})^2[(R^{(0)})^2 - (r^*)^2] r^* \, dr^*. & n\text{-even} \end{cases} \quad (2.12)$$

Note that the conservation of integrated torque states that, to zeroth-order, all the final angular momentum is associated with the planetary torque related to the position of the lens center relative to the system's center of gravity. This is due to the fact that the breakup forces the lenses away from their original center of rotation causing them to acquire planetary torque.

Relation (2.12) can be further simplified to

$$w^{(0)} = \pi (R^{(0)})^6 (n^3 - n)/64 \quad (2.13)$$

which, together with (2.11), gives the desired relationship between n and $R^{(0)}$

$$\boxed{(R^{(0)})^2 = \frac{1}{n^2 - 1}.} \quad (2.14)$$

In terms of n, the nondimensional width is

$$\boxed{w^{(0)} = \frac{n\pi}{64(n^2-1)^2}} \quad (2.15)$$

which completes our solution to the problem. The dependency of $R^{(0)}$ and $w^{(0)}$ on n is shown in Fig. 5 and the energy loss as a function of n is displayed in Fig. 6.

An additional point should be made before the present discussion is completed. It is clear that for each given chain of identical lenses there is only one box from which they could have originated. This does not necessarily mean, however, that a given box can only result in one pattern of eddies. Limited attempts to find additional patterns (i.e. patterns other than a chain of identical lenses) showed that other patterns are impossible but a completely satisfactory answer to this uniqueness question is left as a subject for further

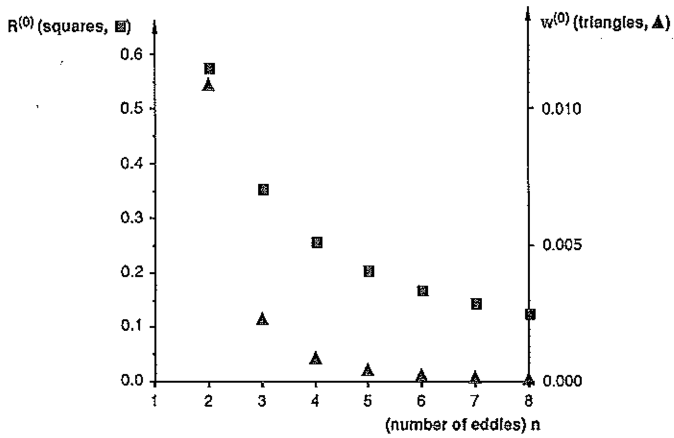

Fig. 5. The size of the final eddies $R^{(0)}$ and the size of the original box $w^{(0)}$ as a function of the number of eddies. The computations are based on the zeroth-order expansion. Note that, due to the impossibility of a noninteger n, there are only sets of sizes that are physically possible.

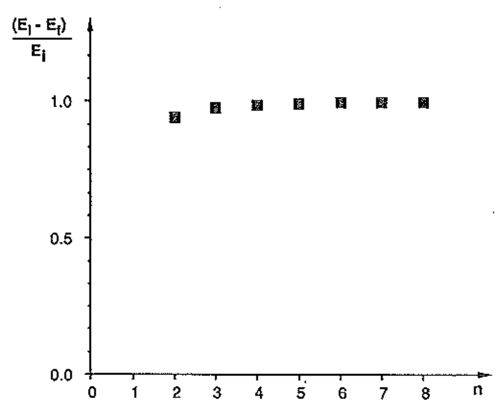

Fig. 6. The zeroth-order relative energy loss $[(E_i - E_f)/E_i]$ as a function of the number of lenses (n). The energy loss is a result of long gravity waves that radiate most of the initial energy away. It is relatively large due to the large initial depth (H).

investigations. It is also important to point out that when the width of the box is not a modal value the eddies are not of uniform size. However, because of the conservation of momentum (2.2), the problem remains symmetrical with respect to both x and y. The details for such nonuniform cases probably requires a numerical solution and is beyond the scope of this study.

A mixed (analytical–numerical) solution for an initial box whose horizontal dimensions are comparable to the deformation radius

In this case $\varepsilon \sim O(1)$ and the solution is obtained as follows. First, (2.5) is solved numerically; then, for a given b, w is eliminated from (2.1) and substituted into (2.4) to

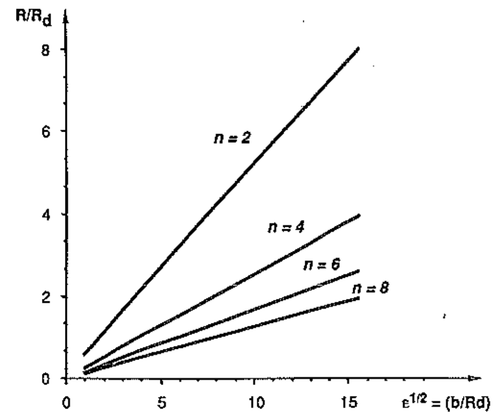

Fig. 7. The mixed (analytical–numerical) solution for the radius of the final eddies (R) as a function of the initial box length ($2b$) and the number of eddies in the chain (n).

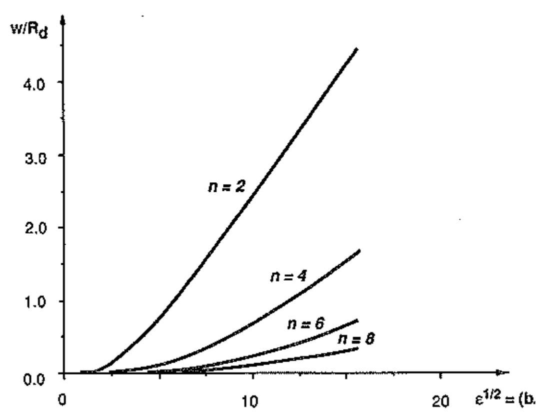

Fig. 8. The same as Fig. 7 but for the initial box width (w).

give a single algebraic equation for R. This equation is again solved numerically verifying that (i) the root corresponds to eddies which do not overlap and (ii) that there is a positive energy loss during the breakup. It turns out that there is only one roof satisfying these conditions.

The solutions for the eddy radius R and the initial box width are shown in Figs 7 and 8. Figure 9 shows the energy loss associated with the collapse. Note that, as in the analytical case, large portions of the energy are radiated away via long gravity waves and that the loss increases as the number of eddies in the chain increases. A comparison between the expanded analytical solution and the mixed (analytical–numerical) solution is shown in Fig. 10, which illustrates that even for $\varepsilon = 1$ the analytical solution is an excellent approximation. This results from the numerical coefficients of the neglected terms which, by themselves (i.e. without the presence of ε) are very small compared to unity.

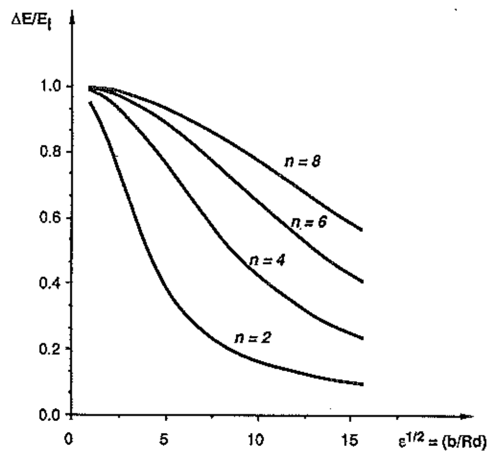

Fig. 9. The relative energy loss ($\Delta E/E_i$) associated with the breakup of a box (whose horizontal dimensions are comparable to the deformation radius) as a function of the box's length and number of eddies in the chain. Losses are based on the mixed (analytical–numerical) solution.

3. THE INTERACTION OF THE CHAIN OF EDDIES WITH THE WALL

The interaction of the single lens with a wall has been examined by Nof (1988), who demonstrated that such eddies leak fluid due to conservation of momentum along the boundary (Fig. 11). In view of this, it is expected that a chain of lenses would have a multi-leakage structure as shown in Fig. 12. Namely, each vortex would leak in the Nof (1988) manner to its neighboring lens which again would leak fluid to the right. Note that, due to the complexity of the lens–wall interaction problem, Nof (1988) has only presented some

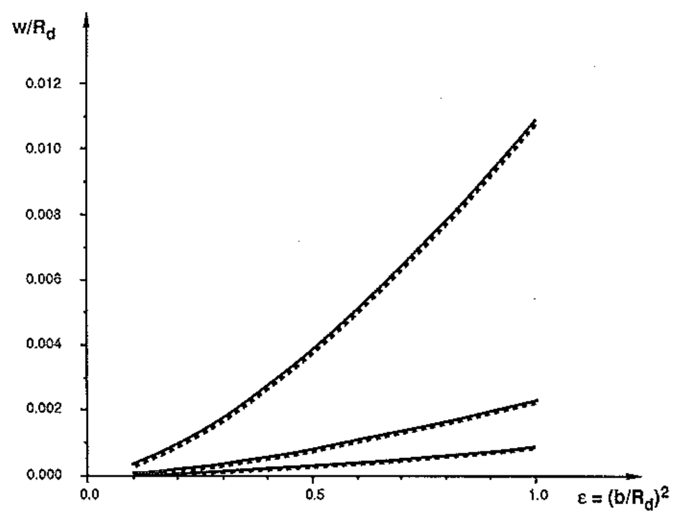

Fig. 10. A comparison between the expanded analytical solution (for boxes whose horizontal dimensions are small compared to R_d) and the mixed solution [for boxes whose dimensions are of $O(R_d)$]. The analytical solution is shown by the solid line and the mixed by the dashed line. Note that the two solutions virtually overlap even for $\varepsilon = 1$. This is due to the numerical coefficients of the neglected terms which are, by themselves, much smaller than unity.

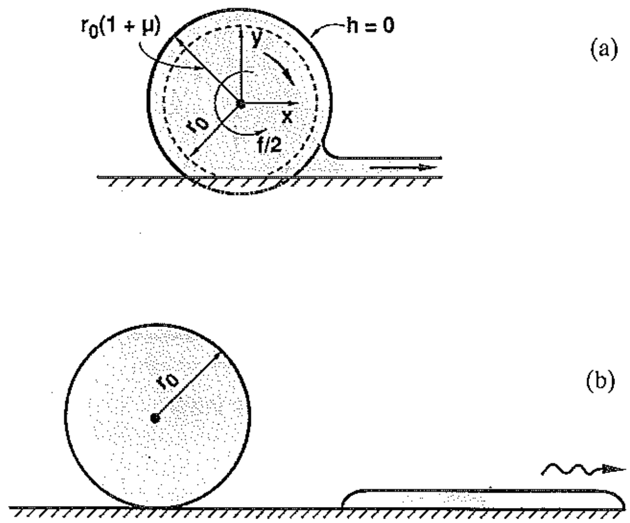

Fig. 11. A sketch of a single lens interacting with a wall (adapted from Nof, 1988). Initially (a), the lens leaks fluid on the right-hand side; the parameter μ measures the wall penetration. Ultimately (b), the fluid that is in direct contact with the wall is entirely removed and we are left with a smaller vortex that is merely 'kissing' the wall and a long narrow strip of the leaked fluid which continues (forever) to propagate to the right (Stern, 1980). The 'wiggly' arrow indicates propagation.

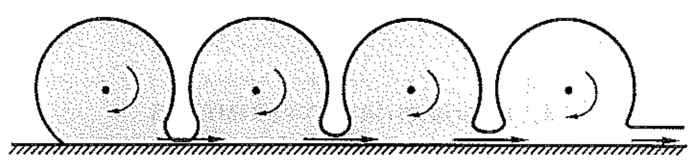

Fig. 12. The expected multi-leakage of a chain of lenses interacting with a wall. Ultimately, the fluid which is in direct contact with the wall will be entirely removed and we will be left with a discrete set of smaller eddies that are 'kissing' the wall and are separated from each other. As in Fig. 11, the leaked fluid forms a long narrow strip.

general considerations and laboratory experiments to support the nonlinear lens leakage idea. We take a similar approach in our present analysis and do not attempt to derive a detailed solution for the chain–wall interaction.

Because of the multiple-leakage considerations, it is expected that the original collapse of the box near the wall (Fig. 3) would also involve a group of eddies that leak together along the wall. In reality, the collapse and the lenses–wall interaction are, of course, taking place simultaneously rather than independently but it is believed (and later verified with the aid of laboratory experiments) that the two-step approach gives the correct qualitative answer to the problem.

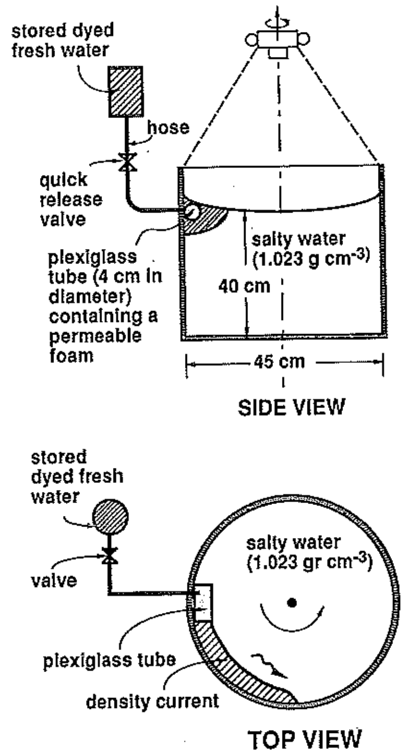

Fig. 13. A sketch of the experimental apparatus. The intermittent density current is established by releasing the stored light fluid for a finite amount of time (approximately 20 s). As before, the 'wiggly' arrow indicates propagation of the density current.

4. LABORATORY EXPERIMENTS

To examine the validity of the foregoing theory a set of qualitative 'kitchen-type' laboratory experiments were performed (Fig. 13). The experiments rely on the establishment and the abrupt termination of a density current advancing along a wall.

Experimental procedures

The apparatus (Fig. 13) was centered to less than ± 1 mm of the table rotation axis. A 35 mm motor-driven camera was mounted on the rotating table and the breakup process was photographed (from above) every few seconds. The tank was rotated counterclockwise at a uniform rotation rate until the system reached a solid rotation (about 30 min). The experiment began with the injection of dyed fresh water through the horizontal tube. After a few seconds the current reached an approximate cross-stream geostrophic balance. Shortly afterwards the injection of fresh water was terminated and the current quickly began breaking up as predicted by the theory. The influence of laboratory friction, though obviously greater than the actual friction in the ocean, was not significant. The breakup

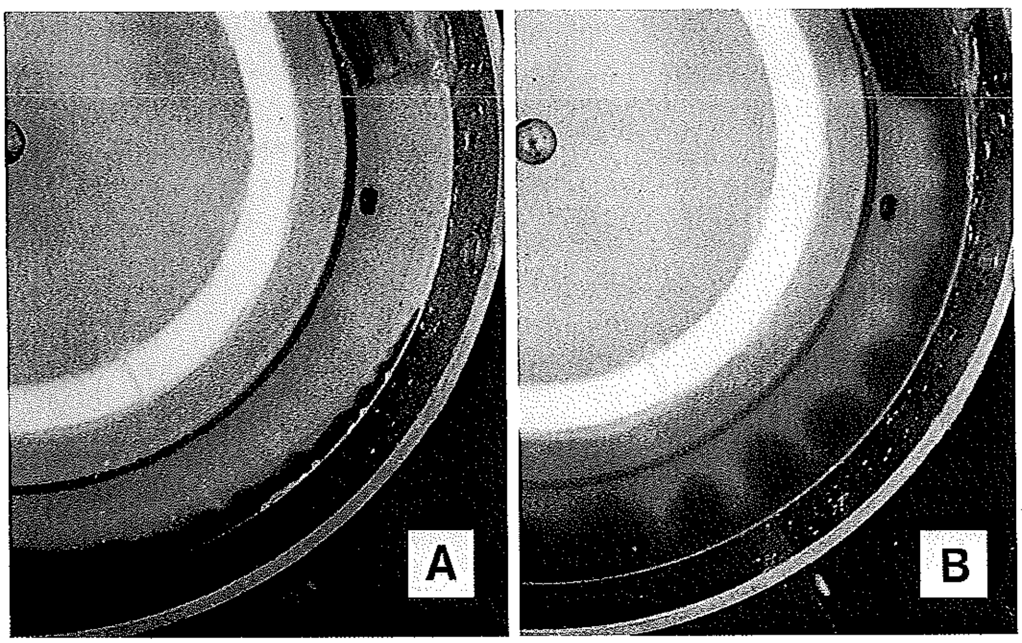

Fig. 14. Two subsequent photographs of an intermittent (laboratory) density current. (A) shows the structure of the current prior to the termination of the flux whereas (B) shows the eddies after the breakup which occurred several rotation periods after the termination. (A) was taken several rotation periods after the initiation of the current (i.e. several rotation periods after the injection began). For clarity, only the right corner of the tank is shown. The tube from which the current originates can be seen on the upper right. The white ring near the center is a reflection of the fluorescent light shining from above. Physical constants: $F = 3.0\ \mathrm{s}^{-1}$; $\Delta\rho/\rho = 0.023$; the mass flux of the initial current was a few cubic centimeters per second. Note that the leakage at the nose continues at all times. The eddies at the front are smaller than those in the rear due to the initial wedge-like shape of the unbroken current.

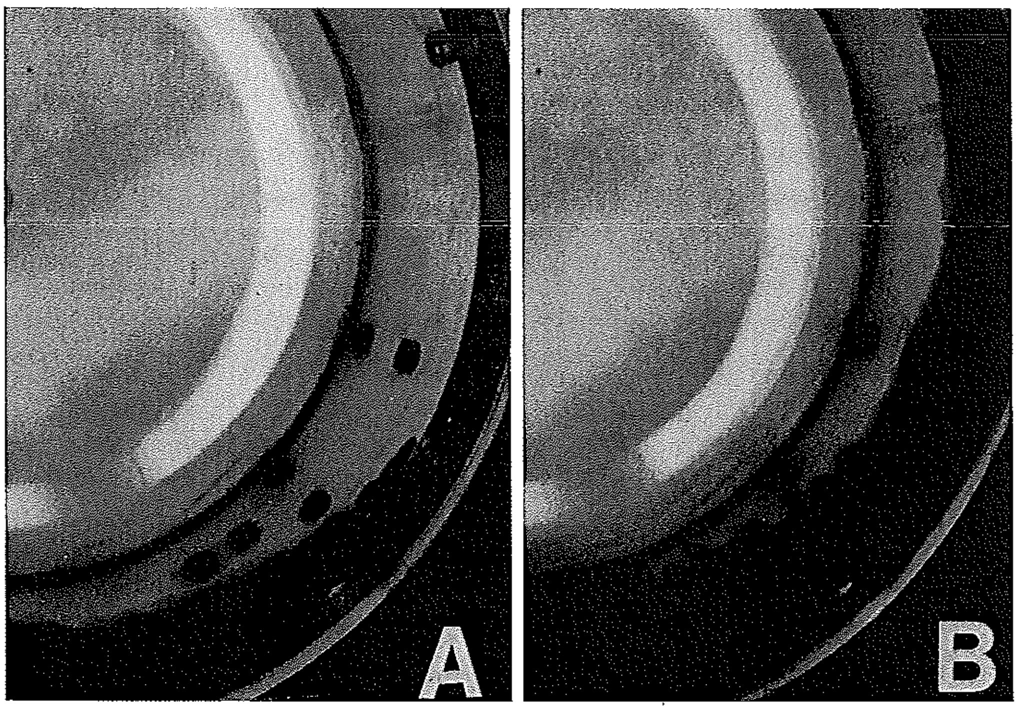

Fig. 15. Two subsequent photographs of a *steady* (i.e. nonintermittent) density current. As in the process shown in Fig. 14, (A) was taken several rotation periods after the initiation of the flow and (B) was taken several rotation periods after (A). Note that the rotation rate, mass flux rate and densities are equivalent to those in the previous experiment.

was observed for about 20 s, whereas the exponential spindown time scale (i.e. the decay to $1/e$ of the initial light fluid depth) was estimated to be much longer—about 10 min.

Interaction with the ambient fluid

A potentially serious difference between our laboratory experiments and the computations presented earlier stems from the fact that the calculations were made for an infinitely deep environmental fluid, whereas the experiment is associated with a finite environmental fluid. Specifically, a density current formed by the injection of light fluid in a finite depth layer will always induce some motion underneath. Because of the depth ratio in our experiment (about 1/8), the role of the induced motion underneath is probably not entirely negligible. It is not expected, however, to be crucial to the processes in question.

Results

Two subsequent photographs of a typical experiment are shown in Fig. 14. The first photograph (A) shows the current prior to the termination of the flux. Waves that are typical to such currents (e.g. STERN, 1980; GRIFFITHS and LINDEN, 1981, 1982; STERN *et al.*, 1982; GRIFFITHS and HOPFINGER, 1983; KUBOKOWA and HANAWA, 1984a,b; GRIFFITHS, 1986) are seen along the front. These waves are not a result of the termination, which has not occurred yet, but rather a consequence of the front. The second photograph (B) shows the eddies generated by the breakup; as the theory predicts (Fig. 12), the eddies leak along the wall. For a comparison, photographs of an equivalent experiment with a current that *was not terminated* (shortly after its formation) are also shown (Fig. 15). Although the front shows the presence of waves, the current did not break up, indicating that it is the *termination* of the flow, rather than the instability to long wave perturbation, that produced the breakup shown in Fig. 14.

It is important to realize that our observed eddies (all of which are connected to each other by small leakages) are different from the high amplitude meanders (in steady flux currents) observed by GRIFFITHS and LINDEN (1981, 1982). As mentioned, the GRIFFITHS and LINDEN (1981, 1982) eddies were produced by long wave instability whereas our eddies were produced by intermittency. Of course, the breakup of the intermittent current also involves some sort of instability because otherwise it would have remained intact. However, this instability (which is not specifically addressed in our study) may not involve long waves and may not be related to the instability of the equivalent steady flow. In other words, the intermittent process produces eddies even if the equivalent steady current (i.e. a current with identical dimensions, flux and density) is *stable* to long wave perturbations. Such stable flows were considered by PALDOR (1983) and appear to be present in the experiment displayed in Fig. 15. Having said that, however, it must also be stated that PALDOR's (1983) stable conditions are not quite realistic (due to an infinite lower layer) and it is possible that the termination of our source flux experiment merely alters the flow sufficiently for it to break up into eddies sooner than it does with a maintained source. Finally, a comment should be made regarding the relationship between our present study and the maintained source experiment of GRIFFITHS and LINDEN (1981). The difference between these studies is reflected in the fact that in the GRIFFITHS and LINDEN (1981) point source case (see their Fig. 10) the mass flux of the connecting eddies along the wall (i.e. the 'leakages') is of the same order as the mass flux circulating within the lenses, whereas in our case the flux of the leakages along the boundary appears to be much smaller than the transport within the eddies.

5. APPLICATION TO THE MEDITERRANEAN OUTFLOW

The foregoing theory might be applicable to numerous outflows and straits because almost all outflows in the ocean are intermittent. For example, Ross (1976) describes current meter measurements in the Denmark Strait taken in 1973 for a period of 36 days. The transport was observed mostly as bursts with a peak of 7 Sv; the average was 2.5 Sv. Another example is the outflow from the Amazon which is also known to be largely time-dependent (Ryther *et al.*, 1967; Gibbs 1970). While these and other outflows are clearly intermittent, the outflow for which there is most data available on both its origin and its final fate is the Mediterranean outflow. For this reason we shall attempt to compare our lenses generation mechanism to the way that the so-called Meddies are formed. Since our model is highly idealized, only qualitative comparison will be made.

Bursts of transport from the Mediterranean into the Atlantic result from atmospheric storms which exert abnormally high pressure over the Mediterranean. They have been noted by Gründlingh* (1981), Garrett (1983), Garrett and Majaess (1984) and Candela *et al.* (1989). A reasonable estimate for the Mediterranean bursts is that, in addition to the usual 1 Sv outflow, there is an excess of about 1 Sv for a period of up to, say, 5–7 days. This gives an outflow excess of $\sim O(10^{12}\text{m}^3)$ during each storm or burst.

Based on salt balances calculations, Armi *et al.* (1989) and Richardson *et al.* (1989) have suggested that it would take about 10 days of regular outflow transport to form a single Meddy. This implies that the initial volume of *one* Meddy is about 10^{12}m^3. Our model suggests that a few (or several) eddies will be formed as a result of a burst of about the same volume (10^{12}m^3) which, as mentioned, is the estimated excess of volume spilled to the Atlantic during a storm over the Mediterranean.

Exactly how many eddies will be formed from this excess volume cannot be determined with our model because we cannot tell what is the appropriate size of the conceptual box which breaks up. However, we can say that the model gives eddies whose volumes are smaller but of the same order as those discussed by Armi *et al.* (1989) and Richardson *et al.* (1989). Note that such a qualitative agreement is the best that one can hope for with such an idealized model. Finally, it should be pointed out that it is also possible that some of the Meddies merge after their formation (e.g. Griffiths and Hopfinger, 1986, 1987; Nof and Simon, 1987; Nof, 1988), thus increasing the volume of each observable vortex.

In summary, it can be said that the primary aim of both our theory and experiment was to examine the formation of lenses from bursts in the transport of outflows. The inviscid generation process is viewed as being the result of a collapse of a box (containing the anomalous fluid) near the walls (Figs 2 and 3). The problem is further simplified by first removing the box offshore (Fig. 4) and examining the collapse there; attention is focused on the final steady state resulting from the collapse. Then, the resulting features—a

* Gründlingh's conclusion regarding bursts of transport relies on his observation of bursts in temperature (e.g. his Fig. 2, record number 20101). It has been suspected that these pulses may not actually be due to transport increase but, rather, due to, say, a meander that passed by his current meter mooring. While this is certainly possible, it appears unlikely since his record shows a rather high degree of asymmetry (in the rise and fall of the temperature), whereas a meander is expected to be more or less symmetrical.

discrete group of eddies—are 'returned' to the coast and the interaction with the wall is examined. Both processes are strongly nonlinear because the amplitudes as well as the Rossby number are of order unity. Although various collapse problems have been solved before with the aid of the potential vorticity conservation and the conservation of volume, the problem at hand contains an additional unknown (the number of eddies) and, therefore, is 'unsolvable' using common techniques. This difficulty is resolved by employing the, rarely used, conservation of integrated angular momentum which provides the closure constraint necessary for the solution.

Our findings are

(1) The offshore collapse produces a discrete set of lenses that are kissing each other. Most of the angular momentum of the final state is associated with the *position* of the eddies, i.e. the fact that the breakup forced the *eddies* away from their original center of rotation causing them to acquire planetary torque.

(2) The relationships between the size of the initial box and the eddies' number and diameter are given in Figs 5, 7 and 8. For boxes whose dimensions are small compared to the deformation radius the results were obtained using solutions that are entirely analytical (Fig. 5), whereas for boxes whose dimensions are comparable to the Rossby radius the solutions are partially numerical (Figs 7 and 8). During the collapse a large portion of the energy is radiated away via long gravity waves (Figs 6 and 9). As expected, the larger the number of eddies resulting from the breakup of the box, the larger is the energy loss.

(3) When a chain of eddies interacts with a wall a chain of leakages is produced (Fig. 12). A detailed solution for this complicated case is beyond the scope of this study. The suggested general structure is based on NOF's (1988) theory and experiment for a single vortex interacting with the wall (Fig. 11).

(4) To examine the validity of the theory (1) and (2) and conclusion (3), a series of simple qualitative laboratory experiments on a rotating table were performed. An intermittent boundary current was created by injecting dyed fresh water near a vertical wall of a container which included salt water (Fig. 13) and then terminating the injection. The experiments clearly illustrate that the intermittent current breaks up into a group of eddies that interact with the wall (Fig. 14) as suggested by our theory (Fig. 12) and that an equivalent current that is not terminated does not break up (Fig. 15).

The above theory and experiments are applicable to various straits and outflows. It is suggested that (i) transport intermittency is a powerful generation mechanism for lenses and (ii) that bursts in transport may explain the generation of Meddies (i.e. the lenses that are generated from the Mediterranean outflow and are frequently observed in the Atlantic Ocean).

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