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The β-induced drift of separated boundary currents

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Abstract—Western boundary currents flow poleward from low latitudes until they ultimately separate from the coast and turn eastward into the ocean interior. The separation is mainly due to either: (i) the variation of the Coriolis parameter with latitude (β) which causes vanishing of the near-wall depth; (ii) vanishing wind stress curl over the ocean interior which forces zero meridional transport; or (iii) opposing currents that flow toward the equator and force the northward flowing currents to turn offshore (AGRA and NOF, *Deep Sea Research I*, **40**, 2259–2282). Here, we focus on the *third* kind of separated currents and show that, due to β, such separated currents *migrate* along the wall. A nonlinear "reduced gravity" one-and-a-half layer model is used to compute the desired migration speed. Solutions of the primitive equations are constructed analytically assuming that the translation rate is steady. It is found that the migration rate along the wall is given by $\beta R_d^2 \cos\alpha/2 \sin\gamma$, where R_d is the Rossby radius, α an angle that measures the inclination of the joint offshore currents relative to the north, and γ is the angle between the axis of the joint offshore currents and the wall. The migration meridional component can be either northward or southward (depending on the inclination of the wall) but the zonal component is always westward. When the separated joint offshore flow is in the east–west direction (i.e. $\alpha = \pi/2$ or $3\pi/2$ so that the separated flow is *zonal*) no migration is taking place. It turns out that the above migration formula is so robust that it also describes the migration rate in a two-and-a-half layer model where one current is allowed to, at least partially, dive under the other. For most separated currents the computed migration rate is a few centimeters per second.

Possible application of this theory to the Confluence zone in the South Atlantic (where significant seasonal movement of the separation latitude has been observed) is discussed.

1. INTRODUCTION

THE separation of western boundary currents has been the subject of numerous investigations during the past 50 years. Most of the relevant studies are cited in the recent articles of CESSI *et al.* (1990), OLSON *et al.* (1988) and AGRA and NOF (1993) and need not be referenced here. For our present discussion it is sufficient to say that, as a result of these studies, it is now understood that separation can be caused by four, apparently independent, processes.

The first is related to the variation of the Coriolis parameter with latitude, which forces the near-wall depth of a northward flowing inertial boundary current to vanish at mid-latitude. The second process is associated with vanishing wind stress curl over the ocean interior, which causes no meridional transport at that particular latitude. The third process is related to the shape of the coastline in the vicinity of the separation latitude, which gives

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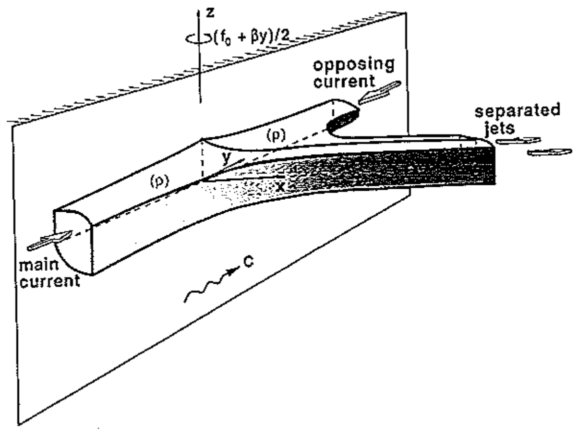

Fig. 1. Schematic diagram of separated jets drifting on a β-plane. The separation is due to an opposing jet with density identical to that of the main western boundary current. The unperturbed ($\beta = 0$) opposing current and main current are related in such a manner that the system is balanced, i.e. it is stationary on an f-plane (AGRA and NOF, 1993). The "wavy" arrow indicates the migration due to β.

the impression that the continent is leaving the current rather than that the current is leaving the coast. All three of these processes have been studied extensively and verified by various techniques. A fourth separation process has been recently proposed (AGRA and NOF, 1993) and is unrelated to either β, the wind field, or the local geography of the detachment region. It is associated with opposing southward flowing boundary currents that force the northward flowing western boundary currents to turn offshore (Figs 1 and 2).

The purpose of our present study is to examine the drift that β imposes on the *fourth* separation process (i.e. to determine how colliding jets which turn offshore are affected by the presence of β). For our basic unperturbed state (i.e. no β), we shall consider what has been referred to by AGRA and NOF (1993) as "balanced currents," i.e. colliding jets that are stationary on an f-plane. Such a stationary system satisfies a particular relationship

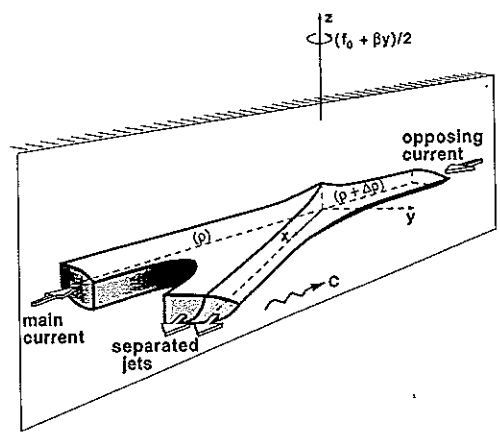

Fig. 2. The same as Fig. 1 except that the opposing current which causes the separation has an intermediate density so that it partially dives under the main current. To illustrate this diving effect the jets are viewed from an angle different than that used in Fig. 1.

between the two colliding currents, which implies that the momentum imparted by one current on the other is balanced. Most currents in the ocean are probably unbalanced, implying that, in addition to the β-induced drift in which we are presently interested, there also will be a drift due to the unbalanced component. As pointed out by AGRA and NOF (1993), this unbalanced drift simply can be thought of as advection because it is associated with the strong current pushing the system toward the weak current. Although this unbalanced drift will be discussed in passing, it is not the focus of the present investigation, which only will be concerned with the effects of β. The computation of the unbalanced drift is not straightforward; consequently, it is the subject of a separate investigation and will be reported elsewhere.

It is rather important to realize that we shall focus on what shall be later referred to as "weak β" corresponding to a situation where the length scale L satisfies $\beta L/f_0 \ll 1$. This distinguishes our analysis from that which could bring in the first separation process where the influence of β is "strong," i.e. the meridional length scale satisfies $\beta L/f_0 \sim O(1)$. Using analytical techniques we shall show that, as expected, the presence of a weak β introduces a long-shore drift of the separation point. The drift is of the order of the long Rossby wave speed and can either be toward or away from the equator depending on the inclination of the coast relative to the north.

To show this, we shall consider the inviscid shallow water equations in a coordinate system tilted in such a manner that one axis is aligned with the coast (Fig. 3); the system translates with the separated currents at an (assumed) steady rate C. We shall integrate the corresponding equations of motion analytically and use a perturbation scheme in $\beta L/f_0$ which, as just mentioned, is taken to be small. This analytical procedure enables us to construct a solution for the translation rate without solving for the complete first-order problem. Two models will be considered. First, we shall look at a "reduced gravity," one-and-a-half layer model where the colliding jets have identical densities (Section 3). We shall then proceed to the more realistic and complicated case of a two-and-a-half layer model where the colliding jets have different densities so that one jet is diving under the other. It turns out that, even though the two-and-a-half layer model is considerably more complicated than the one-and-a-half layer model, the associated drifts of the two models are quite similar.

After presenting the above results, an application of the theory to the Confluence zone in the South Atlantic is discussed (Section 4). In this region the Brazil and Malvinas currents collide and subsequently separate from the coast. The joint currents and the separation point have been observed to migrate along the shore as much as 500 km in a season. It is suggested that this drift is, at least partly, a result of our β-induced migration that can cause comparable displacements.

2. A "REDUCED GRAVITY" ONE-AND-A-HALF LAYER MODEL FOR SEPARATED JETS PERPENDICULAR TO THE COAST

In this section we shall look at two jets with identical densities colliding on a β-plane. For clarity, we begin with a simple special case. Namely, we shall first look at two jets that are not only balanced (i.e. stationary on an f-plane) but also have such relative strengths that they leave the coast at a 90° angle [see Fig. 3 and relation (13) in AGRA and NOF, 1993]. Later on in Section 3 we shall relax this constraint and examine more general jets that leave the coast at any angle. It is important to recognize at this point that, although the

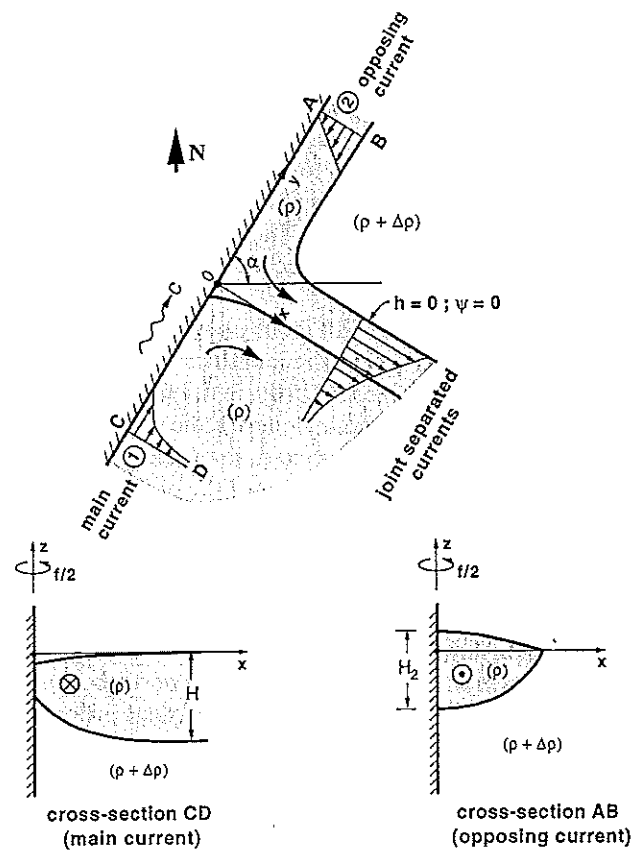

Fig. 3. Schematic diagram of colliding jets with identical densities that translate steadily on a β-plane. This is a special case where the relative strength of the colliding jets is such that the resulting joint offshore flow leaves the coast at a 90° angle, i.e. the unperturbed jets are not only balanced but also satisfy the condition that equation (13) in AGRA and NOF (1993) yields a 90° angle. The x axis is directed along the off-shore streamline that separates the two jets (i.e. the streamline that can be traced back to the wall).

following theory is for currents that are balanced on an f-plane, the balance conditions derived by AGRA and NOF (1993) are not used explicitly. Namely, although the present theory requires that the zeroth-order flow is balanced, the detailed flow solution need not necessarily be that of AGRA and NOF (1993). In this sense, the following analysis is more general than that of AGRA and NOF (1993).

(*i*) *Formulation*

Consider the colliding jets shown in Fig. 3. The coordinates system is oriented in such a way that the x axis represents the separating streamline of the joint offshore currents so that the y axis points along the coast. The system travels at speed C along the coast so that the motion appears to be steady. In such a traveling coordinate system the governing shallow water equations are

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+[f_0+\beta(y\sin\alpha-x\cos\alpha)]\,u+g'\frac{\partial h}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-[f_0+\beta(y\sin\alpha-x\cos\alpha)]\,(v+C)+g'\frac{\partial h}{\partial x}=0 \tag{2}$$

$$\frac{\partial}{\partial x}(hu)+\frac{\partial}{\partial y}(hv)=0, \tag{3}$$

where the notation is conventional and the presence of x and α in the Coriolis parameter f results from the tilt of the coordinates system.

The boundary conditions are

$$u=0; \qquad x=0 \tag{4a}$$

$$(u^2+v^2)=\text{constant along }\psi=0 \tag{4b}$$

$$u\geq 0; \qquad x\to\infty \tag{4c}$$

$$v\to v_2; \qquad h\to h_2; \qquad y\to+\infty \tag{4d}$$

$$v\to v_1; \qquad h\to h_1; \qquad y\to-\infty \tag{4e}$$

where the subscripts "1" and "2" correspond to the upstream regions shown in Fig. 3. The first relation (equation (4a)) reflects the condition that the wall is a streamline whereas the second (equation (4b)) corresponds to an application of the Bernoulli integral along the front where $h=0$. The third (equation (4c)) is a (radiation) condition allowing the fluid to move offshore (but not inshore) and the fourth and fifth (equations (4d) and (4e)) correspond to the known upstream structure of the two approaching jets.

As mentioned, our aim is to determine the migration rate C. Usually, this would require finding the complete solution for the entire problem but we shall see that, in this particular case, C can be determined without solving for the whole field. This will be achieved by integrating the momentum equation along the coast and using a pertubation scheme.

(ii) *Integrated momentum*

To obtain the migration rate (equation (1)) will be integrated over the region shown in Fig. 4. Integration in z and the introduction of a streamfunction ψ defined by

$$\frac{\partial\psi}{\partial y}=-uh; \qquad \frac{\partial\psi}{\partial x}=vh, \tag{5}$$

gives

$$\frac{\partial}{\partial x}(huv)+\frac{\partial}{\partial y}(hv^2)+[f_0+\beta\,(y\sin\alpha-x\cos\alpha)]\left(-\frac{\partial\psi}{\partial y}\right)+\frac{g'}{2}\frac{\partial}{\partial y}(h^2)=0. \tag{6}$$

We proceed by integrating (equation (6)) over the area S,

$$\int_S\!\!\int\left[\frac{\partial}{\partial x}(huv)+\frac{\partial}{\partial y}(hv^2)\right]\mathrm{d}x\mathrm{d}y-\int_S\!\!\int[(f_0+\beta(y\sin\alpha-x\cos\alpha)]\frac{\partial\psi}{\partial y}\,\mathrm{d}x\mathrm{d}y$$
$$+\frac{g'}{2}\int_S\!\!\int\frac{\partial}{\partial y}(h^2)\,\mathrm{d}x\mathrm{d}y=0,$$

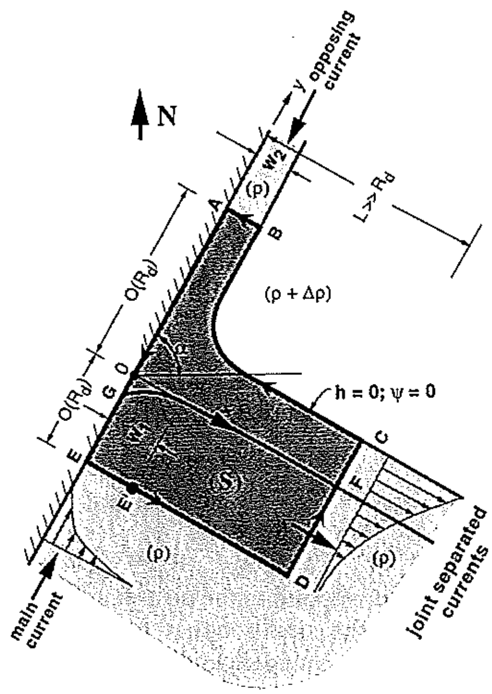

Fig. 4. The integration areas (S) for the perpendicular jets shown in Fig. 3. The x axis is pointing along the streamline separating the joined offshore jets and the y axis is pointing along the coast. Cross-sections AB and DE are situated several deformation radii away from the axis so that the flows there are one-dimensional. Cross-section CD is situated a distance L (much greater than the deformation radius but smaller than the radius of the earth) away from the wall. E is situated several deformation radii away from the wall (i.e. $w_1 > R_d$).

which, using Stokes' theorem, can be simplified to:

$$\oint huv\mathrm{d}y - \oint hv^2\mathrm{d}x + \oint (f_0 - \beta x \cos\alpha)\,\psi\mathrm{d}x + \int_S\int\left[\frac{\partial}{\partial y}(-\psi\beta y\sin\alpha) + \psi\beta\sin\alpha\right]\mathrm{d}x\mathrm{d}y + \frac{g'}{2}\oint h^2\,\mathrm{d}x = 0. \quad (7)$$

Here, arrowed circles indicate counterclockwise integration along the boundary of S. Noting that all along the boundary at least one of the three variables h, u and v vanishes (i.e. h vanishes along CB, u vanishes along BA and ED, and v vanishes along AE and DC), we conclude that the first integral is zero. Using the Stokes' theorem again equation (7) now can be written as

$$\beta\sin\int_S\int\psi\mathrm{d}x\mathrm{d}y + \oint\left\{-hv^2 + [f_0 + \beta(y\sin\alpha - x\cos\alpha)]\,\psi - \frac{g'h^2}{2}\right\}\mathrm{d}x = 0. \quad (8)$$

Leaving this y momentum integral aside for a moment, we now turn to the geostrophic flow at section ED (situated away from the joint streamline GF) and multiply the corresponding x momentum equation by h,

$$[f_0 + \beta(y \sin\alpha - x \cos\alpha)]\,(v + C)h - \frac{g'}{2}\frac{\partial}{\partial x}(h^2) = 0. \tag{9}$$

This equation also can be expressed as

$$(f_0 + \beta y \sin\alpha)\frac{\partial \psi}{\partial x}\frac{\partial}{\partial x}(\beta x \psi \cos\alpha) +$$

$$\beta\psi \cos\alpha + [f_0 + \beta(y \sin\alpha - x \cos\alpha)]Ch - \frac{g'}{2} - \frac{\partial}{\partial x}(h^2) = 0,$$

which upon integration in x from the wall ($x = 0$) to a point away from the wall gives,

$$(f_0 + \beta y \sin\alpha)\,(\psi - \psi_w) - \beta x \cos\alpha\psi + \beta \cos\alpha \int_0^x \psi \mathrm{d}x$$

$$+ (f_0 + \beta y \sin\alpha)C \int_0^x h\mathrm{d}x - \beta \cos\alpha C \int_0^x xh\mathrm{d}x \; - g'(h^2 - h_w^2)/2 = 0, \tag{10}$$

where we are, of course, free to choose the absolute value of ψ_w.

We now return to the integrated y momentum equation (8). First, we note that the line integral vanishes along CB because both ψ and h vanish there. Second, we substitute equation (10) into equation (8) to find,

$$\beta \sin\alpha \int_S\!\!\int \psi \,\mathrm{d}x\mathrm{d}y + \int_B^A \{-hv^2 + [f_0 + \beta(y \sin\alpha - x \cos\alpha)]\psi - g'h^2/2\}\,\mathrm{d}x$$

$$+ \int_E^D \left[-hv^2 - \beta \cos\alpha \int_0^x \psi\mathrm{d}x - (f_0 + \beta y \sin\alpha)C \int_0^x h\mathrm{d}x\right.$$

$$\left. + \beta \cos\alpha C \int_0^x xh\mathrm{d}x + (f_0 + \beta y \sin\alpha)\psi_w - \frac{g'}{2}h_w^2\right]\mathrm{d}x = 0, \tag{11}$$

which will be shortly scaled and simplified.

(iii) The migration rate

The propagation speed C can be extracted from equation (11) by two methods, both of which are presented below. The first (and simplest) one is simply to look at the order of magnitude of the various terms and ignore the small terms. The second is more rigorous (and more complicated) and involves a formal expansion. As should be the case, of course, both methods give the same answer. The reader is warned in advance that the scaling presented below is not at all trivial and requires careful consideration. Although an attempt has been made to make the presentation as clear as possible, for some, it might be easier to skip this part and return to it later.

Before presenting our first method it is recalled that: (a) as is typically the case with boundary currents, within the jets the speed is $O(g'H)^{1/2}$, where H is the undisturbed depth away from the wall (see Fig. 3); this speed decays away from the jet's axis. Far from the jet's axis, the speed is $O(\beta R_d^2)$, because the speed away from the jet's axis is of the same order as the system drift speed, βR_d^2. Note that, without β, there is no drift so that the speed away from the jet's axis vanishes; (b) the chosen integration boundary (Fig. 4) is such that x ranges from zero to L where L is much greater than R_d (but much smaller than the radius of the earth) whereas y ranges from zero to a distance that is of $O(R_d)$. For convenience, L can be taken to be the intermediate length scale, $(f_0 R_d^2/\beta)^{2/3}$, although, as we shall see, this is not essential.

(*A*) *Method* I. Since the x length scale (L) is much greater than the y scale (R_d), and the zeroth order terms ($\beta = 0$) cancel each other, it is clear that the dominant terms in equation (11) are those that involve β, C and a double integration in x. Noting that, in addition, $C \sim O(\beta R_d^2)$ and $\beta L \ll f_0$, we find that equation (11) can be approximated by,

$$\int_E^D \left[\beta \cos\alpha \int_0^x \psi \, dx + f_0 C \int_0^x h \, dx\right] dx = 0. \tag{12}$$

Since ED is situated several deformation radii away from the front ($h = 0$), ψ and h can be replaced by $g'H^2/2f_0$ and H, respectively. Together with equation (12), this gives,

$$\boxed{C = -\frac{\beta R_d^2 \cos\alpha}{2},} \tag{13}$$

which is our desired expression for the migration speed.

Relationship (equation (12)) essentially implies that, since L is large, the integrated momentum associated with the two-dimensional interaction area near the wall is small relative to that of the joint offshore jets. Hence, the only momentum which matters is that of the one-dimensional mutual flow; the dynamical role of the coastline is merely to orient the migration in a fixed pre-determined course. Relation (13) states that perpendicular joint flow does not drift if the wall is meridional ($\alpha = \pi/2$) and that the drift is always "westward" because when $\alpha > \pi$ the y axis is directed toward the equator instead of the pole. Note that the westward component of the drift is $-\beta R_d^2 \cos^2\alpha/2$ *which is not identical to the long Rossby-wave speed.*

(*B*) *Method II—a formal expansion*. We begin by defining the following nondimensional variables,

$$\left.\begin{array}{lll}
\psi^* = \dfrac{\psi}{g'H^2/f_0}; & x^* = x/L; & y^* = y/R_d \\
h^* = h/H; & v^* = v/(g'H)^{1/2}; & C^* = C/\beta R_d^2 \\
R_d = \dfrac{(g'H)^{1/2}}{f_0}; & L = \left(\dfrac{f_0 R_d^2}{\beta}\right)^{1/3}; & \varepsilon = \dfrac{\beta L}{f_0} = \left(\dfrac{\beta R_d}{f_0}\right)^{2/3} \\
\dfrac{R_d}{L} = \varepsilon^{1/2}; & w_{1,2}^* = \dfrac{w_{1,2}}{R_d}, &
\end{array}\right\} \tag{14}$$

where w_2 is the width of the opposing jet and w_1 is a measure of the main jet width (the distance EE'). For convenience and convenience alone, we chose L to be the intermediate length scale (which for $R_d \sim 40$ km is about 200 km); as alluded to before, L could be any length scale greater than R_d but smaller than the radius of the earth. Note that, as mentioned, away from the jet axis, $v \sim O(\beta R_d^2)$ and not of $O(g'H)^{1/2}$.

In terms of these new scaled variables, the integrated momentum equation (11) is

$$\varepsilon^{2/3}\sin\alpha \iint_S \psi^*\,dx^*\,dy^* + \int_{\varepsilon^{1/2}w_2^*}^{0} \{-h^*(v^*)^2 + [1+\varepsilon^{3/2}(y^*\sin\alpha - \varepsilon x^*\cos\alpha)]\psi^*$$
$$-(h^*)^2/2\}\,dx^* + \int_0^{\varepsilon^{1/2}w_1^*} -h^*(v^*)^2\,dx^* + \int_{\varepsilon^{1/2}w_1^*}^{D} -h^*(v^*)^2\,dx^*$$
$$+\int_0^D \left\{-\varepsilon\cos\alpha\int_0^{x^*}\psi^*\,dx^* - (\varepsilon+\varepsilon^{5/2}y^*\sin\alpha)C^*\int_0^{x^*}h^*\,dx^* + \varepsilon^2 C^*\cos\alpha\int_0^{x^*}x^*h^*\,dx^*\right.$$
$$\left. -(h_w^*)^2/2 + (1+\varepsilon y^*\sin\alpha)\psi_w^*\right\}dx^* = 0, \quad (15)$$

where, as mentioned, w_2^* is the nondimensional width of the opposing jet and w_1^* is a measure of the main jet width. Note that the integral of $h^*(v^*)^2$ from E to D has been split into two parts because it involves two different scales, i.e. within the jet $v^* \sim O(1)$ but away from the jet $v^* \sim O(\varepsilon^{3/2})$. We proceed by expanding the unknown variables:

$$\left.\begin{aligned} v^*(x^*,y^*,\varepsilon) &= v^{(0)}(x^*,y^*) + \varepsilon^{3/2}v^{(1)}(x^*,y^*) + \ldots \\ h^*(x^*,y^*,\varepsilon) &= h^{(0)}(x^*,y^*) + \varepsilon^{3/2}h^{(1)}(x^*,y^*) + \ldots \\ \psi^*(x^*,y^*,\varepsilon) &= \psi^{(0)}(x^*,y^*) + \varepsilon^{3/2}\psi^{(1)}(x^*,y^*) + \ldots \end{aligned}\right\} \quad (16)$$

where the zeroth-order state corresponds to balanced jets on an f-plane [i.e. as described by Agra and Nof (1993), these separated jets are stationary on an f-plane] and the first-order terms correspond to the perturbations due to β. The power of ε in the first-order expansion has been taken in such a way that, as required, the perturbed speeds are of $O(\beta R_d^2)$.

Substituting equation (16) into equation (15) and noting that the zeroth-order terms correspond to the momentum associated with the f-plane jets which is automatically satisfied, we find after some quite tedious algebra that the leading order [$O(\varepsilon)$] equation is

$$\int_E^D \left[-\cos\alpha\int_0^{x^*}\psi^{(0)}\,dx^* - C^*\int_0^{x^*}h^{(0)}\,dx^*\right]dx^* = 0, \quad (17)$$

which leads to our previously derived relationship (equation (13)) as should be the case.

3. A ONE-AND-A-HALF AND A TWO-AND-A-HALF LAYER MODEL FOR SEPARATED JETS WHICH ARE NOT NECESSARILY PERPENDICULAR TO THE COAST

Consider now the more general balanced case of two opposing jets whose relative strength is such that the resulting joint offshore flow leaves the coast at any angle (i.e. $0 \le \gamma \le \pi$ as shown in Fig. 5). Again, we choose the x axis so that it is aligned with the streamline separating the jets constituting the merged offshore flow. Since now the offshore jet is not perpendicular to the wall, the y axis is no longer parallel to the wall. Hence, the main difference between the present case and the perpendicular offshore flow discussed earlier in Section 2 is that now the long-shore migration C has also a component pointing in the x direction. In view of this, the relevant equations are

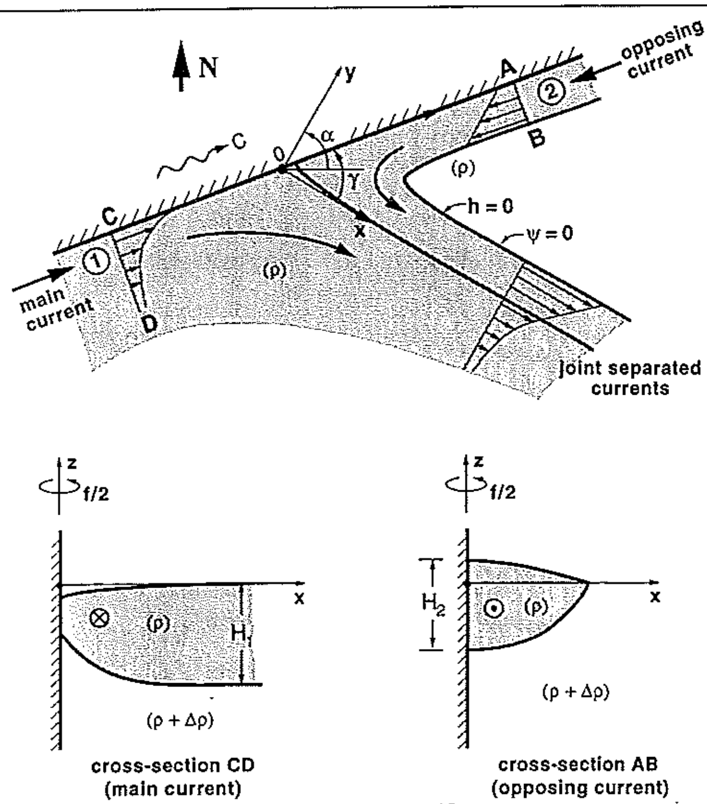

Fig. 5. The same as Fig. 3 but for jets whose resulting joint flow leaves the coast at an angle γ that is not neessarily perpendicular to the coast. This is a generalization of the special case discussed earlier.

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + [f_0 + \beta(y\sin\alpha - x\cos\alpha)]\,(u + C\cos\gamma) + g'\frac{\partial h}{\partial y} = 0 \tag{18}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - [f_0 + \beta(y\sin\alpha - x\cos\alpha)]\,(v + C\sin\gamma) + g'\frac{\partial h}{\partial x} = 0 \tag{19}$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0. \tag{20}$$

As before, we begin by integrating the y momentum equation over the area shown in Fig. 6. In contrast to the previous (perpendicular) case, however, the integral now involves a force on the wall (because the y axis is no longer parallel to the coast) and an additional term due to the x component of C. One ultimately finds that the appropriate integrated momentum equation equivalent to equation (8) is

$$\beta\sin\alpha\int_S\int\psi\,\mathrm{d}x\mathrm{d}y + \oint[-hv^2 + [f_0 + \beta(y\sin\alpha - x\cos\alpha)]\psi - g'h^2/2]\,\mathrm{d}x$$
$$+ C\cos\gamma\iint[f_0 + \beta(y\sin\alpha - x\cos\alpha)]\,h\,\mathrm{d}x\mathrm{d}y = 0. \tag{21}$$

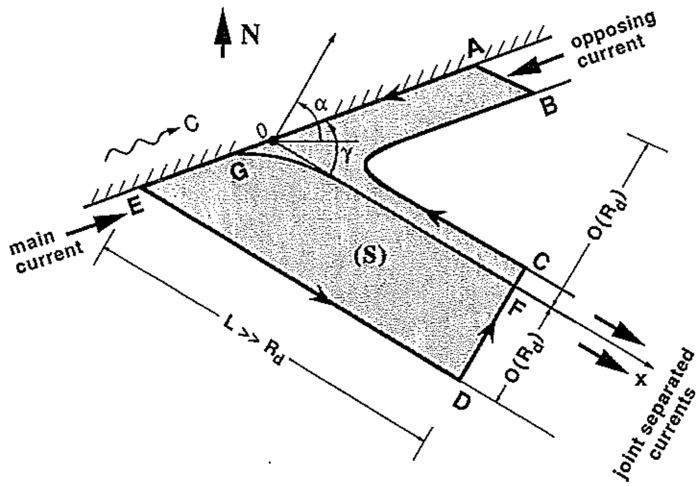

Fig. 6. The integration areas (S) for opposing jets (with identical densities) whose resulting joint flow leaves the coast at an angle (γ) which is not necesarily 90°.

Again, we temporarily leave this equation and look at the x momentum equation (19). We note that away from the wall (i.e. outside the jet's core equation where $x \gg R_d$) the velocities u and v are of $O(\beta R_d^2)$. Hence for this area, the nonlinear terms in equation (19) are of $O(\beta R_d/f_0)^2$ and can be dropped so that the flow is approximately geostrophic and the equation can be integrated in x,

$$[f_0 + \beta(y \sin\alpha - x\cos\alpha)]\psi + \beta\cos\alpha\int_0^x \psi \mathrm{d}x + f_0 C\sin\gamma\int_0^x h\mathrm{d}x + g'h^2/2$$
$$+ \beta y\sin\alpha\, C\sin\gamma\int_0^x h\mathrm{d}x - \beta\cos\alpha\, C\sin\gamma\int_0^x xh\mathrm{d}x - \beta x\psi_w\cos\alpha = 0. \quad (22)$$

As before, we define ψ to be zero along the front where $h = 0$.

We now proceed in the same manner that the perpendicular jets were treated, i.e. equation (21) and equation (22) are nondimensionalized and an expansion equivalent to equation (16) is used. It turns out that, even though there is a force on the wall in the present case, this force is small compared to those associated with the offshore jets because it is associated with a length scale that is of $O(R_d)$ whereas the offshore forces are associated with the much greater length scale L. One ultimately finds,

$$\int_E^D\left[-\cos\alpha\int_0^{x^*}\psi^{(0)}\,\mathrm{d}x^* - C^*\sin\gamma\int_0^{x^*}h^{(0)}\,\mathrm{d}x^*\right]\mathrm{d}x^* = 0, \quad (23)$$

which gives,

$$\boxed{C = -\frac{\beta R_d^2}{2}\frac{\cos\alpha}{\sin\gamma} + O\left(\frac{\beta^2 R_d^3}{f_0}\right)} \quad (24)$$

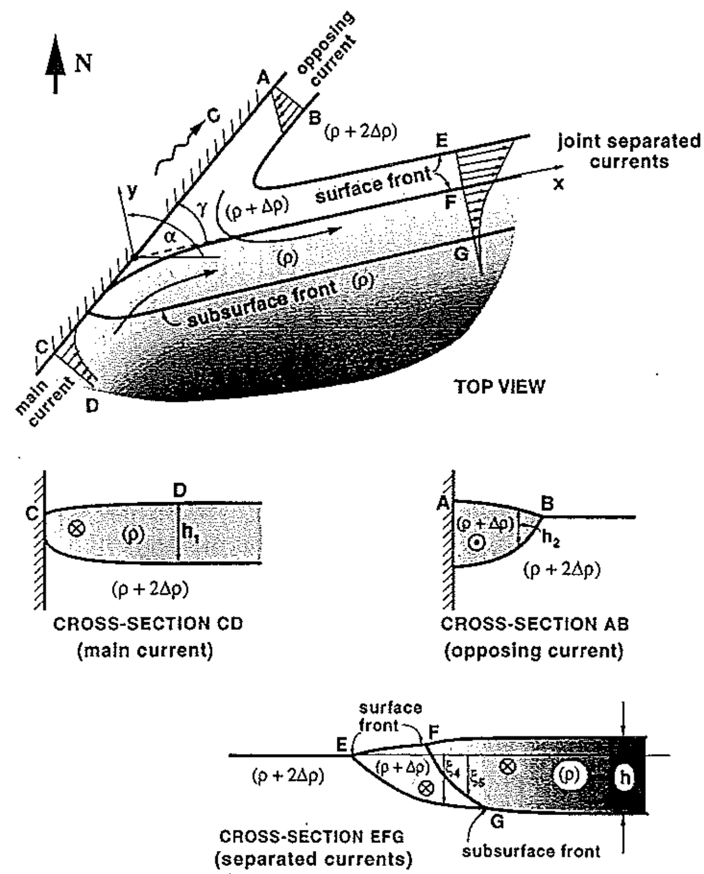

Fig. 7. A detailed schematic diagram of joint separated currents of different densities on a β-plane shown earlier in Fig. 2. As in the balanced two-and-a-half layer model of AGRA and NOF (1993), the opposing jet has an intermediate density so that it partially dives under the separated main current.

Three comments should be made with regard to equation (24). First, when $\gamma = 90° - \alpha$ the wall is zonal so that the migration is forced to be zonal as well. Under such conditions, equation (24) reduces to $-\beta R_d^2/2$, which is identical to the results found by NOF and DEWAR (in press) for free mid-ocean jets, as should be the case. Namely, as expected, in this particular case, the wall does not play any dynamical role so that the fluid behaves as if the wall did not exist. Second, as in the perpendicular jets case, all separated jets move "westward" (because $0 \leq \gamma \leq \pi$), but the westward component, $\beta R_d^2 \cos^2\alpha/2 \sin\gamma$, is again not identical to the westward propagation rate of long Rossby waves, βR_d^2. Third, it is not entirely clear what happens when α and γ are time-dependent although it is expected that, under such conditions, the migration will be time-dependent too.

Consider now the two-and-a-half layer model shown in Fig. 7 and the integration area displayed in Fig. 8. This situation is more realistic and resembles more closely the oceanic situation (e.g. OLSON *et al.*, 1988) because the colder opposing current is now allowed to, at least partially, dive under the warmer main current. To derive the solution for this

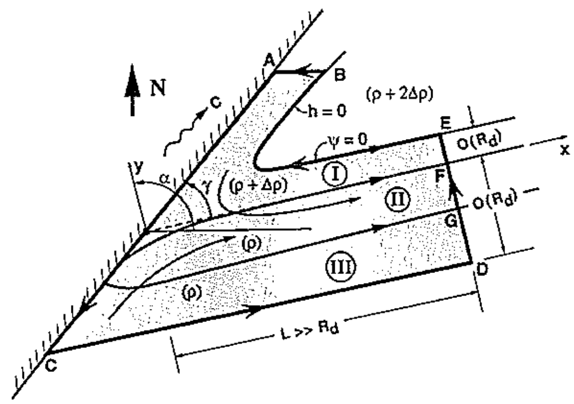

Fig. 8. The integration area for the two-and-a-half layer model shown in Fig. 7.

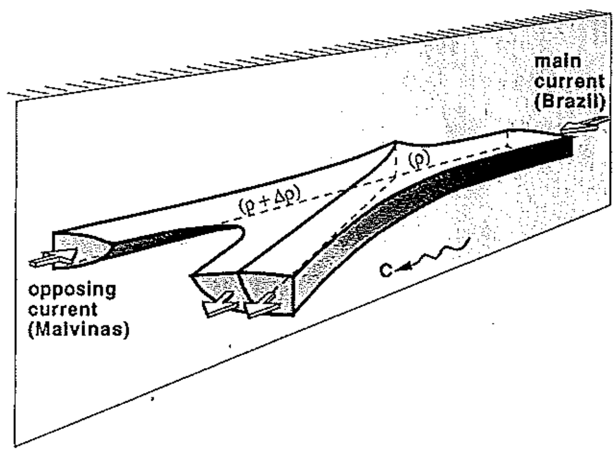

Fig. 9. Schematic diagram of our two-and-a-half layer model application to the confluence zone in the South Atlantic. Here, the Brazil Current is associated with the southwestward flowing main current whereas the Malvinas Current is associated with the opposing northeastward current.

particular case, we proceed as previously and integrate the appropriate y-momentum equations over the area. After elimination of the interactive pressure forces by adding the integrated momentum equation for the various regions and some tedious algebra (taking into account the special balance condition corresponding to the relationship between the upstream depth of the main and opposing currents) one ultimately finds,

$$C = \beta R_d^2 \cos\alpha/\sin\gamma + O(\beta^2 R_d^3/f_0). \tag{25}$$

This relationship is essentially identical to (24) because R_d, the deformation radius of the main current, is still defined as $(g'H)^{1/2}/f_0$, but the density difference between the main current and the layer underneath is now $2\Delta\rho$ instead of $\Delta\rho$.

4. APPLICATION AND CONCLUSION

We shall now consider an application of our two-and-a-half layer model to the Confluence zone in the South Atlantic (Figs 9 and 10). Before beginning our detailed

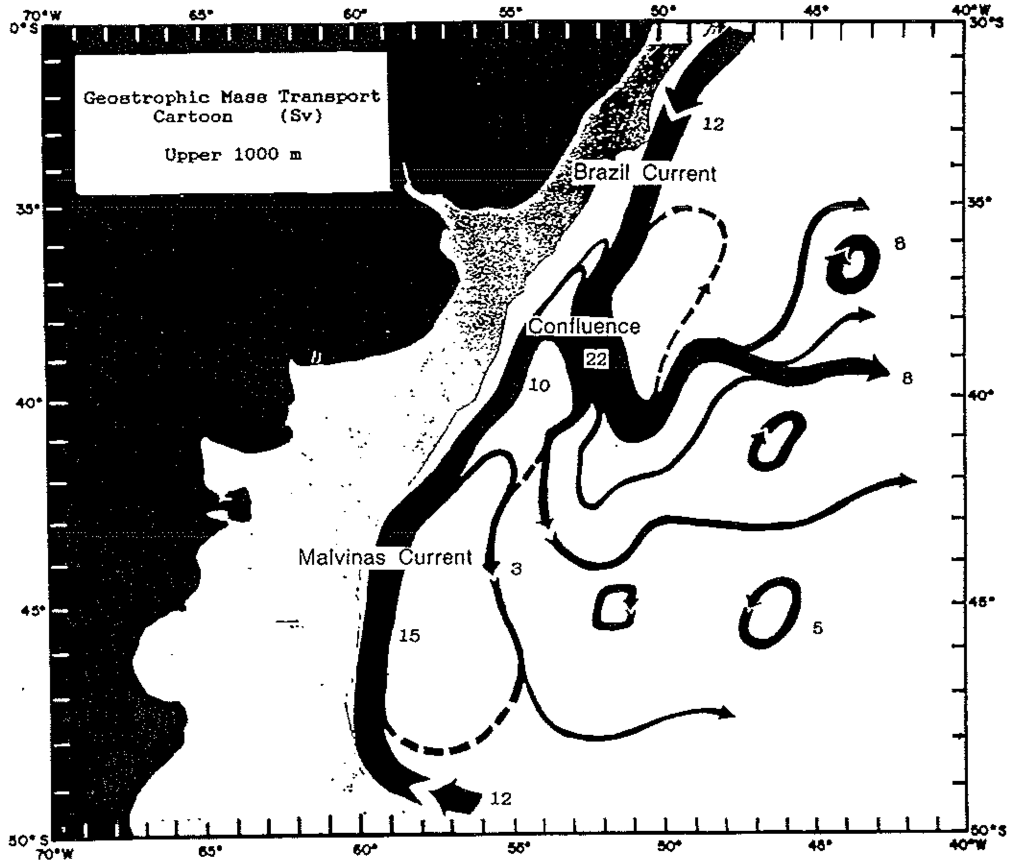

Fig. 10. Schematic of geostrophic time averaged mass transport of the upper 1000 m in the western Argentina Basin based on the hydrographic data collected within the Confluence Project. The transports given are in Sverdrups (1 Sv $= 1 \times 10^6$ m^3 s^{-1}). Reproduced from Confluence Principal Investigators (1990).

discussion, the reader is again referred to Agra and Nof (1993), where an application of the balanced f-plane solution to the Brazil–Malvinas currents system is discussed. The reader also is referred to Olson *et al.* (1988) and the references given there where the observational aspects of the problem are discussed. The variability of the wind pattern that drives the system in the South Atlantic is given in Large and Van Loon (1989).

We begin our application by noting that, according to the structure displayed in Fig. 10, the appropriate angles in the Confluence zone of the South Atlantic are $\gamma \sim 60°$ and $\alpha \sim 150°$. (To convert the situation in the South Atlantic to a comparable structure in the northern hemisphere the reader may wish to look at Fig. 10 as if it is viewed from the back of the page and upside down and then compare it to Fig. 5.) A typical depth for the main (Brazil) current is 700 m and an appropriate value for the reduced gravity $g\Delta\rho/\rho$ is roughly 0.01 m s^2. Together with $f_0 \sim 10^{-4}$ s^{-1}, $\beta \sim 2 \times 10^{-11}$ s^{-1} m^{-1} and the appropriate angles mentioned above, equation (25) gives

$$C \approx 1.2 \text{ km day}^{-1}$$

as a typical migration speed toward the south pole. The observations suggest that the actual oceanic drift is comparable. According to OLSON *et al.* (1988), the seasonal variations of the separation latitude can be as much as 400 km corresponding to a migration rate of about 3 km day^{-1} (for, say, 140 days). This comparison, is, of course, inconclusive, suggesting only that the β-induced drift may be large enough to be significant.

Although the observed migration rate and the computed β-induced drift are of the same order suggesting that our proposed mechanism is potentially an important oceanic process, it is clear that β is *not the only* mechanism that causes the separation point to drift. This is so because the observations suggest a seasonal oscillatory movement of the separation point (OLSON *et al.*, 1988) whereas the theory suggests a steady southwest migration. The difference between the actual drift and the β-induced drift is probably due to the fact that the actual currents are not balanced. As pointed out by AGRA and NOF (1993), unbalanced currents [i.e. currents whose near-wall depths do not satisfy $H_2 = \sqrt{2}\, H_0$, where H_0 and H_2 are the depths of the (light) main and (heavier) opposing current] must form a drifting system even on an f-plane because the momentum imparted by one current on the other cannot be balanced with a stationary joint offshore flow.

Most currents in the ocean are probably unbalanced since it is highly unlikely that they will satisfy the condition mentioned above. Consequently, most currents colliding on an f-plane form a joint offshore flow that drifts toward the weaker current. Scaling suggests that the inviscid (unbalanced) f-plane drift can perhaps be greater than our newly predicted β-induced drift (a few centimeters per second). However, since this drift is caused by unequal Bernoulli pressure along the walls, viscosity (which usually brings the new wall speed down to zero) is expected to reduce this speed dramatically. These aspects are presently being investigated and will be reported elsewhere. The idea that a β-induced drift of a few centimeters per second is important is supported by OLSON *et al.* (1988) observations which suggest a comparable total migration rate of approximately 4 cm s^{-1}. Note that the (unbalanced) f-plane drift is probably indirectly related to the wind field as it is this forcing mechanism that drives the currents in the first place.

Overall it can be said that, just as with mesoscale eddies which are forced westward by both β and advection by mean currents, separated joint offshore flows resulting from colliding jets drift along the coast due to both β and advection resulting from the unbalanced component. This particular kind of advection associated with the unbalanced drift is presently being studied as a part of a separate investigation.

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